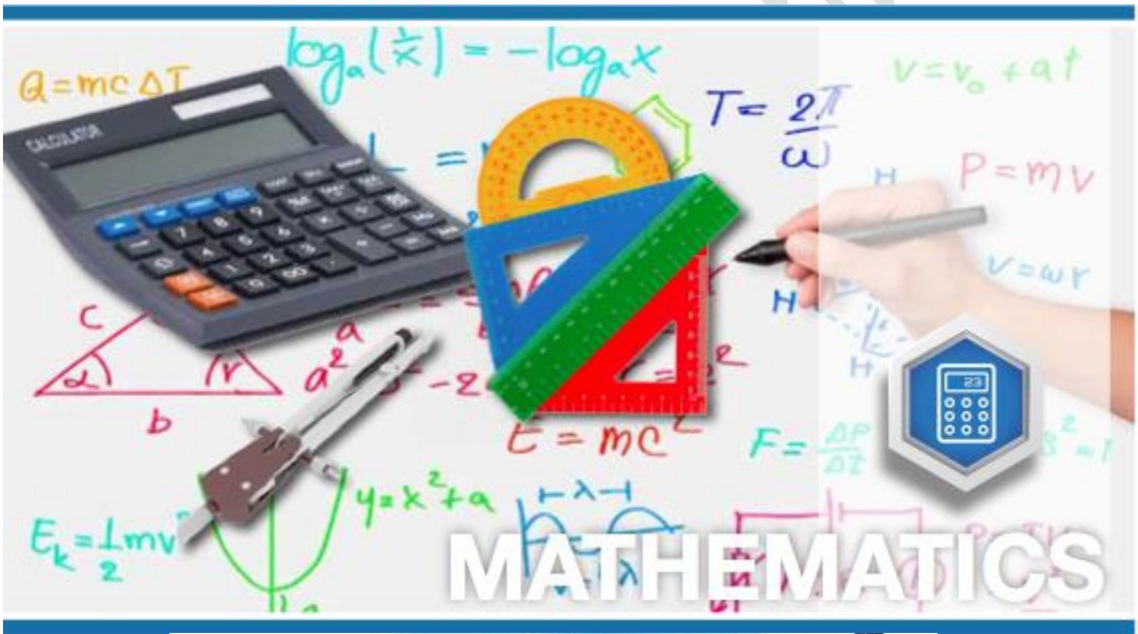
D – Mathematics

Notes Booklet



Contents	www.2Easy2Learn.Com	Page

3. Syllabus content at a glance

All candidates will study the following themes or topics:

Theme or topic	Theme or topic
1. Number	22. Sequences
2. Set language and notation	23. Variation
3. Squares, square roots, cubes and cube roots	24. Graphs in practical situations
4. Directed numbers	25. Graphs of functions
5. Vulgar and decimal fractions and percentages	26. Function notation
6. Ordering	27. Coordinate geometry
7. Standard form	28. Geometrical terms
8. The four operations	29. Geometrical constructions
9. Estimation	30. Similarity and congruence
10. Limits of accuracy	31. Symmetry
11. Ratio, proportion, rate	32. Angles
12. Percentages	33. Loci
13. Use of an electronic calculator	34. Measures
14. Time	35. Mensuration
15. Money	36. Trigonometry
16. Personal and small business finance	37. Vectors in two dimensions
17. Algebraic representation and formulae	38. Matrices
18. Algebraic manipulation	39. Transformations
19. Indices	40. Probability
20. Solutions of equations and inequalities	41. Categorical, numerical and grouped data
21. Graphical representation of inequalities	42. Statistical diagrams

Assessment objectives

The two assessment objectives in Cambridge O Level Mathematics are:
AO1 Mathematical techniques
AO2 Applying mathematical techniques to solve problems

Relationship between assessment objectives and components

The table shows the assessment objectives as an approximate percentage of each component and as an approximate percentage of the overall Cambridge O Level Mathematics qualification.

Component	AO1 (%)	AO2 (%)	Weighting of component in overall qualification (%)
Paper 1	55–65	35–45	50
Paper 2	28–38	62–72	50
Weighting of AO in overall qualification	40–50	50–60	

Assessment at a glance

All candidates take two papers: Paper 1 and Paper 2.
Each paper may contain questions on any part of the syllabus and questions may assess more than one topic.

Paper 1	**2 hours**
Paper 1 has approximately 25 short answer questions. Candidates should show all working in the spaces provided on the question paper. Essential working must be shown for full marks to be awarded. **No calculators are allowed for this paper.** 80 marks This paper will be weighted at 50% of the total qualification.	

Paper 2	**2 hours 30 minutes**
Paper 2 has approximately 11 structured questions. Candidates should answer **all** questions. **Electronic calculators may be used and candidates should have access to a calculator for this paper.** Candidates should show all working in the spaces provided on the question paper. Essential working must be shown for full marks to be awarded. 100 marks This paper will be weighted at 50% of the total qualification.	

Additional materials for examinations:

For both Paper 1 and Paper 2, candidates should have these geometrical instruments:

- a pair of compasses
- a protractor
- a ruler.

Tracing paper may be used as an additional material for both of the written papers.

For Paper 2, candidates should have an electronic calculator – see below for details.

Use of calculators:

Paper 1 – the use of all calculating aids is prohibited.

Paper 2 – all candidates should have a **silent** electronic calculator. A scientific calculator with trigonometric functions is strongly recommended. Algebraic or graphical calculators are not permitted.

The General Regulations concerning the use of electronic calculators are contained in the *Cambridge Handbook*.

Unless stated otherwise within an individual question, three-figure accuracy will be required. This means that four-figure accuracy should be shown throughout the working, including cases where answers are used in subsequent parts of the question. To earn accuracy marks, premature approximation should be avoided.

In Paper 2, candidates are encouraged to use the value of π from their calculators. Otherwise, they should use the value of π given on the front page of the question paper as 3.142 to three decimal places.

Unit -1

Number

Key points

- **Natural Numbers:**
  The natural numbers are those used for counting (as in "there are six coins on the table") and ordering (as in "this is the third largest city in the country"). The set of natural numbers is represented mathematically by the set: $\boldsymbol{N} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}, \dots \dots \dots \dots . .\}$

- **Whole Numbers:**
  Whole numbers are positive numbers, including zero, without any decimal or fractional parts. They are numbers that represent whole things without pieces.
  The set of whole numbers is represented mathematically by the set: W= {0, 1, 2, 3, 4, 5...}

- **Integers (ℤ):**
  Integers are like whole numbers, but they **also include negative numbers** ... but still no fractions allowed. $Z = \{\dots \dots \dots \dots -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots \dots \dots \dots \dots\}$

- **Even Numbers:**
  An even number is an integer which is "evenly divisible" by two. This means that if the integer is divided by 2, it yields no remainder. Zero is an even number because zero divided by two equals zero. Even numbers are either positive or negative.
  $E = \{0, 2, 4, 6, 8, \dots \dots \dots \dots \dots\}$

- **Odd Numbers:**
  The integer (not a fraction) that cannot be divided exactly by 2. The last digit is 1, 3, 5, 7 or 9. Example: -3, 1, 7 and 35 are all odd numbers. $O = \{1, 3, 5, 7, \dots \dots \dots . .\}$

- **Prime Number:**
  A prime number is a number which has only two different factors 1 and the number itself.
  Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19,…………

- **Composite Number:**
  A composite number is a number which has more than two different factors. A composite number can be expressed as the product of two or more prime numbers.
  Composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, ………………………………

- **The number 1:**
  The number 1 is neither a prime nor a composite number because it has only one factor.

- **Square Number:**
  A **square number** or **perfect square** is an integer that is the **square** of an integer; in other words, it is the product of some integer with itself. For example, 9 is a **square number**, since it can be written as 3×3.

- **Square Root:**
  Finding the square root of a number is the inverse operation of squaring that number.
  Remember, the square of a number is that number times itself.
  e.g. 3 squared is 9, so a **square root of 9 is 3.**

- **Cube Numbers:**
  The result of using a whole number in a multiplication three times.
  Example: $3 \times 3 \times 3 = 27$, so 27 is a cube number.

- **Cube Roots:**
  The inverse of cubing a number is finding the cube roots.
  Example: the cube of 4 is 64 ($4^3 = 4 \times 4 \times 4 = 64$) and cube root of 64 is 4 $\left(\sqrt[3]{64} = 4\right)$.

- **Rational Number ($\mathbb{Q}$):**
  A rational numbers are counting numbers, integers and number which can be expressed as fractions$\left(0, 1, -30, \frac{3}{4}, -25\frac{1}{3}, \frac{22}{7}\right)$. A rational number can be expressed as a terminating or recurring decimal. Examples of rational numbers are: $\frac{22}{7}$, complete square roots $\left(e.g. \sqrt{16} = 4\,, \sqrt{144} = 12\right)$, complete cube roots $\left(e.g\ \sqrt[3]{8} = 2,\ \sqrt[3]{125} = 5\right)$

- **Irrational Number:**
  A non-terminating and non-recurring decimal. Examples:
  Incompetesquare roots $\left(\text{e.g.} \sqrt{2}, \sqrt{80}\right)$, Incomplete cube roots $\left(\sqrt[3]{25}, \sqrt[3]{121}\right)$, $\boldsymbol{\pi}$

- **Surds:**
  When we can't simplify a number to remove a square root (or cube root etc) then it is a surd.
  Example: $\sqrt{2}$ (square root of 2) can't be simplified further so it is a **surd.**
  Example: $\sqrt{4}$ (square root of 4) **can** be simplified (to 2), so it is **not a surd.**

- **Real Numbers($\mathbb{R}$):**
  The union of rational and irrational numbers.

- **Directed Numbers:**
  Directed numbers can be positive as well as negative. The sign indicates a direction. Example:
  -10 m form sea level means 10 m below sea level and
  $+10$ m shows 10 m above sea level.

- **Absolute value of a number:**
  The absolute value or modulus $|x|$ of a real number x is the non-negative value of x without regard to its sign. For example, the absolute value of 3 is 3, and the absolute value of -3 is also 3. The absolute value of a number may be thought of as its distance from zero. To show the absolute value of something, we put "|" marks either side (they are called "bars").
  Examples: $|-5| = 5$, $|7| = 7$, $|8-3| = 5$, $|3-8| = 5$

- **Factors of a Numbers:**
  Factors are numbers we can multiply together to get another number: Example: 5 and 7are factors of 35, because $5 \times 7 = 35$. A number can have MANY factors! For example factors of 12 are 1, 2, 3, 4, 6, 12. Its mean that factor of a number divides that number completely.

- **Multiples of a number:**

  Multiples are what we get after multiplying the number by an integer (not a fraction).

  Examples: Multiples of 3 are 6, 9, 12, 15, 18 and so on

  $3 \times 2 = 6,\quad 3 \times 3 = 9,\quad 3 \times 4 = 12,\quad 3 \times 5 = 15,\quad 3 \times 6 = 18$

  Note: 2 and 3 are factors of 6 and 6 is multiple of 2 as well as 3.

- **Prime Factorisation:**
  The process of expressing a composite number as the product of prime factors is called prime factorisation.

- **Index Notation:**
  In general, a× a× a× a× ……×a is written as a^n and is read as a to the power of n.

- **Highest Common Factor(HCF):**
  The largest of the common factors of two or more numbers is called the Highest Common Factor(HCF) of the numbers.

  How to find HCF by using prime factors in index notation:
  Select **ONLY** common factors with least power.
  Example: Find HCF of $2^3 \times 3^3 \times 5^2 \, and \; 2^2 \times 3^5$

  $$\mathbf{HCF = 2^2 \times 3^3 = 4 \times 27 = 108}$$

- **Lowest Common Multiple (LCM)**: The smallest of the common multiples of two or more numbers is called the Lowest Common Multiple (LCM) of the numbers.

  How to find LCM by using prime factors in index notation:
  Select common factors with greatest power and also uncommon factors.
  Example: Find LCM of $2^3 \times 3^3 \times 5^2 \, and \; 2^2 \times 3^5$
  $\mathbf{LCM = 2^3 \times 3^5 \times 5^2}$

- **Relation between HCF and LCM of two numbers**:
  Product of two numbers is equal to product of LCM and HCF.
  Consider two numbers p and q. $\mathbf{p \times q = LCM \times HCF}$

- **Addition of Integers:** Same sign integers are added and result carries common sign.

  Examples: **i)** $(+2) + (+5) = +7$ ii) $(-2) + (-5) = -7$

- **Subtraction of Integers:** Different sign integers are subtracted and result carries the sign of larger number.

  Examples: **i)** $(+2) + (-5) = -3$ ii) $(-2) + (+5) = +3$

- **Multiplication of Integers:**

  When same sign integers are multiplied then the result carries positive sign.

  Examples: **i)** $(+2) \times (+5) = +10$ ii) $(-2) \times (-5) = +10$

  When different sign integers are multiplied then the result carries negative sign.

Examples: **i)** $(+2) \times (-5) = -10$ ii) $(-2) \times (+5) = -10$

- <u>**Division of Integers:**</u>

  When same sign integers are divided then the result carries positive sign.

  Examples: **i)** $(+14) \div (+2) = +7$ ii) $(-14) \div (-2) = +7$

  When different sign integers are divided then the result carries negative sign.

  Examples: **i)** $(+14) \div (-2) = -7$ ii) $(-14) \div (+2) = -7$

- <u>**Estimation:**</u> It is a method of making an informed guess at the size of a measurement or other value.
- <u>**Approximation:**</u> It is expressing a measurement or other value to a convenient or sensible degree of accuracy.
- <u>**Estimation and Approximation**</u>

  Rules of rounding a number to a given number of significant figures:

a) Count the given number of significant figures from left to right, starting with the first non-zero figure. Include one extra figure for consideration.
b) If the extra figure is less than 5, drop the extra figures and all other following figures. Use zeros to keep the place value if necessary.
c) If the extra figure is 5 or more, add 1 to previous figure before dropping the extra figure and all other following figures. Use zeros to keep the place value if necessary.

Rules for determining the number of significant figures:

a) The following figures in a number are significant:
   i. All non-zero figures.
   ii. All zeros between significant figures.
   iii. All zeros at the end of a decimal.
b) The following figures in a number are not significant:
   i. All zeros at the beginning of a decimal less than 1.
   ii. All zeros at the end of a whole number may or may not be significant. It depends on how the estimate is made.

Unit-2

Indices and Standard Form

<u>**Key Points**</u>

Indices: An index (plural **indices**) or power indicates how many of a certain number or a variable is multiplied together, for example $p^5 = p \times p \times p \times p \times p$.

Laws of Indices

i) $a^m \times a^n = a^{m+n}$ ii) $a^m \div a^n = a^{m-n}$ iii) $(a^m)^n = a^{m\times n}$ iv) $a^0 = 1$

v) $(a \times b)^m = a^m \times b^m$ vi) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ vii) $a^{-n} = \frac{1}{a^n}$

viii) $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ ix) $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ x) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Unit-3

Algebraic Expression and Manipulation

Key Points

Expansion of Algebraic Expressions

- $a(b+c) = ab + ac$
- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a+b)(a-b) = a^2 - b^2$

Factorization of Algebraic Expressions

- **Taking out a Common factors**: To factorise an algebraic expression, always look for a common factor. If there is a common factor, then take it out and use the difference of two squares formula

- **Making Groups (factorize four terms):**

  $x^2 - 2x + 5x - 5 = 2x(x-1) + 5(x-1) = (x-1)(2x+5)$

  This does two things. First, the four terms are swapped around and regrouped if necessary, then the pairs of terms are factorized in such a way that a common factor results. In this example, (x-1) is now a common factor, so that if the factorise command is used one more time the expression will be fully factorised.

- **Difference of perfect squares** $a^2 - b^2 = (a+b)(a-b)$

- **Algebraic Identities**

  $a^2 + 2ab + b^2 = (a+b)^2 = (a+b)(a+b)$

  $a^2 - 2ab + b^2 = (a-b)^2 = (a-b)(a-b)$

- **Factorizing Quadratic Expression (Mid-Term Break)**

  There is no simple method of factorizing a quadratic expression, but with a little practice it becomes easier. One systematic method, however, is as follows:

  A "quadratic" is a polynomial that looks like "$ax^2 + bx + c$", where "a", "b", and "c" are just numbers. For the easy case of factoring, you will find two numbers that will not only multiply to equal the constant term "c", but also add up to equal "b", the coefficient on the x-term. For instance:

Evaluation of Algebraic Expressions

- The process of replacing the variables in an expression with the numerical values and simplifying it is known as evaluating an algebraic expression.
- Order of operation is used to evaluate an algebraic expression.

Parenthesis | Exponents | Multiplication | Division | Addition | Subtraction

1. Perform the operations inside a parenthesis first
2. Then exponents
3. Then multiplication and division, from left to right
4. Then addition and subtraction, from left to right

Addition and Subtraction of Algebraic Expressions

"**Like terms**" are terms that contain the **same variables** raised to the **same power**.

- $3x^2$ and $7x^2$ are **like terms.**
- $-8x^2$ and $5y^2$ are **not like terms**, because the variable is not the same.
- We can only add or subtract like terms.
- Simplify $13x + 7y - 2x + 6a = 11x + 13y$

Algebraic Fractions

- Algebraic fractions have properties which are the same as those for numerical fractions, the only difference being that the numerator (top) and denominator (bottom) are both algebraic expressions.
- Sometimes a little more work is necessary before an algebraic fraction can be reduced to a simpler form.

Addition of Algebraic Fractions

- Addition (and subtraction) of algebraic fractions proceeds in exactly the same manner as for numerical fractions.

Subject of the Formula

$$C = \pi d$$

Subject: C Rule: multiply π by *diameter*

In above formula

- The variable on the left, is known as the **subject**: What you are trying to find.
- The formula on the right, is the **rule**, that tells you how to calculate the subject.
- So, if you want to have a formula or rule that lets you calculate d, you need
- to make d, the subject of the formula.
- This is changing the subject of the formula from C to d.

Unit-4

Solution of Equations and Simultaneous Equations

Key Points

Linear Equations in One Unknown

- A linear equation in one unknown is an equation in which the unknown appears only to the first power.
- If the unknown is x, then the only operations that involve x are multiplication or division by a number and addition or subtraction of numbers.
- That is, a linear equation in x does not contain x in the denominator of a fraction, it does not contain a root of x or a power of x other than the first power.

For example,

- $2(x - 3) + 5x = 7 - x$ is a linear equation in x, but
- $3x^2 + 5 = 2x$ is not since it has an x^2 in it.

- To solve linear equations, you add, subtract, multiply and divide both sides of the equation by numbers and variables, so that you end up with a single variable on one side and a single number on the other side.

Fractional Equations with Numerical and Linear algebraic Denominators;

Steps to solving algebraic fractional equations:

- Step 1: Find the least common denominator of the fractions.
- Step 2: Multiply by the least common denominator on both sides of the equation
- Step 3: Reduce the fraction and solve.

Methods to solve simultaneous linear equations in two unknown

There are three methods (**in GCE O level Syllabus**) to solve simultaneous linear equations.

a) Elimination method b) Substitution method c) Graphical Method

Elimination Method

To solve the simultaneous equations, make the coefficients of one of the variables the same value in both equations. Then either add the equations or subtract one equation from the other (whichever is appropriate) to form a new equation that only contains one variable. This is referred to as **eliminating the variable.**

Solve the equation thus obtained. Then substitute the value found for the variable in one of the given equations and solve it for the other variable. Write the solution as an **ordered pair**.

Substitution Method

To solve the simultaneous equations, find the value of y in terms of x (or vice versa) for one of the two equations and then substitute this value into the other equation.

Quadratic Equation

- A quadratic equation is an equation where the highest power of x is x^2., so it is an equation of the form $\mathbf{ax^2 + bx + c = 0}$. There are various methods of solving quadratic equations, as shown below.
    - Factorization
    - Completing the Square Method
    - Quadratic Formula
    - Completing the Square Method

Completing the Square Method

Some quadratics are fairly simple to solve because they are of the form "something-with-x squared equals some number", and then you take the square root of both sides. An example would be:

$(x - 4)^2 = 5$

$x - 4 = \pm\sqrt{5}$ $\quad x = \pm\sqrt{5} \pm 4$ $\quad x = \pm\sqrt{5} + 4$ or $\quad x = \pm\sqrt{5} - 4$

Quadratic Formula:

The Quadratic Formula uses the "a", "b", and "c" from "$ax^2 + bx + c$", where "a", "b", and "c" are just numbers; they are the "numerical coefficients". The Formula is derived from the process of completing the square, and is formally stated as $\mathbf{x = \frac{-b \pm\sqrt{b^2-4ac}}{2a}}$

Unit-5

Everyday Mathematics

Key points

- **Percentage:** A percentage is fraction whose denominator is 100 and we use % sign to represent. A percentage can be converted to a fraction by dividing its value by 100.

- **Ratio:** A ratio expresses a relationship between two quantities of the same kind. It is usually expressed as the fraction of first quantity over the second. To find the ratio of two quantities, the both quantities must have same unit. A ratio has no unit.
- **Rate:** A rate expresses a relationship between two quantities of different kinds.
- **Speed**: The rate of change of distance is called speed. $\text{speed} = \frac{\text{distance}}{\text{time}}$
- **Average Speed:** Average speed of a moving object is given by the formula:

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time}}$$

- **Conversion of Units:**

$$1\text{ km} = 1000\text{ m} \quad \text{and} \quad 1\text{ h} = 3600\text{ s}$$

$$x\ \frac{\text{km}}{\text{h}} = x \times \frac{1000}{3600}\ \frac{\text{meters}}{\text{seconds}} \qquad y\ \frac{\text{m}}{\text{s}} = y \times \frac{3600}{1000}\ \frac{\text{km}}{\text{h}}$$

- **Simple Interest:** $I = \frac{PRT}{100}$, $A = P + I$

  I = simple interest in currency, P = principal amount in currency

  R = interest rate in percentage (rate per annum), T = time in years, A = total amount

- **Compound Interest:** $A = P\left(1 + \frac{R}{100}\right)^n$

P = principal amount in currency R = interest rate in percentage (rate per annum),

A = total amount, n = number of years

- **Profit and Loss asercentage of Cost/Sale Price**

  $\text{Profit} = \text{S.P} - \text{C.P}$ $\qquad$ $\%\text{age Gain} = \frac{\text{Gain}}{\text{C.P}} \times 100$

  $\text{Loss} = \text{C.P} - \text{S.P}$ $\qquad$ $\%\text{age loss} = \frac{\text{loss}}{\text{C.P}} \times 100$

  Where C.P = cost price and S.P = selling price
- **Discount:**

  $\text{Discount} = \text{Marked Price} - \text{Sale Price}$ $\qquad$ $\%\text{age Dis} = \frac{\text{Dis}}{\text{M.P}} \times 100$
- **Limits of accuracy:**

  Number recordings are not always exact, and in some cases they may be rounded up. When a number has been recorded to a certain accuracy - for instance, the nearest 1cm or the closest 10, you can work out its highest and lowest possible values according to the limits of accuracy provided. These outcomes are often referred to as an upper or lower bound.

  Upper Bound (UB): The top end of the range is called upper bound.

Lower Bound (LB): The bottom end of the range is called lower bound.

Addition: Greatest Possible value = UB1 + UB2

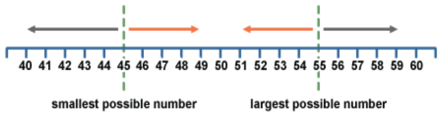
Subtraction: Greatest Possible value = UB1 − LB2 Least Possible value = LB1 − LB2

Multiplication: Greatest Possible value = UB1 × UB2 Least Possible value = LB1 × LB2

Division: Greatest Possible value = $\frac{\text{UB1}}{\text{LB2}}$ Least Possible value = $\frac{\text{LB1}}{\text{UB2}}$

Example: The number of people on a bus is given as 50, correct to the nearest 10. What is the lowest and highest possible number of people on the bus?

Solution: The number 50 is correct to the nearest 10.the number of people on the bus can be any number between 45 and 54



Exponential Growth or Exponential Decay. (IGCSE Syllabus)

Many real world phenomena can be modeled by functions that describe how things grow or decay as time passes. Examples of such phenomena include the studies of populations, bacteria, the AIDS virus, radioactive substances, electricity, temperatures and credit payments, to mention a few.

Any quantity that grows or decays by a fixed percent at regular intervals is said to possess exponential growth or exponential decay. It is given by relations:

Growth : $\boldsymbol{y = a\,(1 + r)^x}$

Decay : $\boldsymbol{y = a\,(1 - r)^x}$

a = initial amount before measuring growth/decay

r = growth/decay rate (often a percent)

x = number of time intervals that have passed

Unit-6

Variation

Key Points

- **Direct Variation:**
  If y varies directly as x, then $y \propto x$ and $y = kx$ where k = constant
- **Inverse Variation:**
  If y varies inversly as x, then $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$ where k = constant

Unit-7

Matrices

Key Points:

- **Matrix:**
  A matrix is an array of numbers.

  $$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

  Row

  Column

- **Square Matrix**: Numbers of rows and columns are equal

Example: $A = \begin{bmatrix} 1 & 5 \\ 10 & 4 \end{bmatrix}$ 2 rows and 2 columns

- **Row Matrix:** It has only one row.

Example: $D = \begin{bmatrix} 7 & -10 & 9 \end{bmatrix}$

- **Column Matrix:** It has only one column.

Example: $C = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$

- **Order of Matrix:**
  The order of the matrix m × n refers to a matrix having m rows across and n columns down.

Examples:

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 & -1 & 4 \\ 6 & 3 & 2 \end{bmatrix}$ $C = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ $D = \begin{bmatrix} 7 & -10 & 9 \end{bmatrix}$

Order = 2 × 2 Order = 2 × 3 Order = 2 × 1 Order = 1 × 3

- **Null Matrix:**
  A zero/null matrix is a matrix whose all elements are zero. It is denoted by O

Examples:

$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $O = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $O = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$

- **Equal Matrices:**
  Two matrices are equal if order of both matrices are same and values of respective elements are also equal.

  If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$, then a=w, b=x, c=y, d=z

- **Identity Matrix:**
  An identity matrix is a square matrix whose diagonal elements are 1 and other elements are zero.
  An identity matrix is denoted by I.

  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **Note:** **A I = I A = A**

- **Addition and Subtraction of Matrices**:

  Two matrices A and B can be added together or subtracted from each other only when A and B are of the same order. In addition or subtraction, the corresponding elements are added or subtracted.

Example1:

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} a+w & b+x \\ c+y & d+z \end{bmatrix}$

$\begin{bmatrix} p & q \\ r & s \end{bmatrix} - \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} p-w & q-x \\ r-y & s-z \end{bmatrix}$

Example2:

$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 8 & 20 \end{bmatrix} = \begin{bmatrix} 1+4 & -1+4 \\ 2+8 & 3+20 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 10 & 23 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ 8 & 20 \end{bmatrix} = \begin{bmatrix} 1-4 & -1-5 \\ 2-8 & 3-20 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ -6 & -17 \end{bmatrix}$

- **Additive Identity**

  O is a null matrix , $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, such that A + O = O + A = A

  Example : Suppose $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

  $O + A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

- **Additive Inverse:**

  Example: Suppose $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, its additive inverse is $B = \begin{bmatrix} -2 & -3 \\ -1 & -4 \end{bmatrix}$,

  such that A + B = O $A + B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ -1 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

- **Multiplication of Matrix by a scalar, k:**

  If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, Then $kA = k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$

- **Multiplication of Matrices:**

  Multiplication of matrices is only possible if columns of first matrix are equal to rows of second matrix.

  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$

  $AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} aw+by & ax+bz \\ cw+dy & cx+dz \end{bmatrix}$

- **Multiplicative Identity**

  I is a identity matrix, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, such that A×I = I ×A = A

  Example: Suppose $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

  $A \times I = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

- **Multiplicative Inverse**

  If A is matrix , then A^{-1} is its multiplicative inverse

  If and only if $A A^{-1} = I$, where I is an identity matrix. $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

  If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

  $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

  $= \frac{1}{det.A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

- **Determinant Of Matrix:**
  If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the determinant of A,
  $|A| = ad - bc$

- **Non-Singular Matrix**

  If $|A| \neq 0, then\ A\ is\ a\ non\ singular\ matrix.$

- **Singular Matrix**
  If $|A| = 0$, then A is a singular matrix, i. e. then inverse of A does not exist.
- **Inverse Matrix:**

  If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the inverse matrix is
  $A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
  $= \frac{1}{det.A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Unit-8

Sets and Venn Diagrams

<u>Key Points</u>

- <u>Set:</u> A group or collection of objects is called a set. The individual objects in a set are called the member or elements of the set.
- A set is <u>define</u> by

a) Listing its elements e.g. A={1, 2, 3, 4}
b) Describing the elements e.g. A={x : x is a teenager} B={x : x < 8}
c) Drawing a Venn diagram

- <u>Finite Set:</u> A set which has a definite number of elements

  A={1, 2, 3} B={x : x is an integer, 1 < x < 15}

- <u>Infinite Set:</u> A set which contain uncountable number of elements.

  A={1, 2, 3, 3,………..} B={x : x is a natural number}

- <u>Empty Set / Null Set :</u> An empty set contains no elements. It is denoted by { } or ∅
- <u>Equal Sets:</u> Two sets are equal if they have exactly the same elements.
- <u>Universal Set:</u> The set which contains all the sets in a discussion. It is denoted by ε.

  A={1,2,3} B={2, 4, 5, 6} C={1, 3, 5} ε={1,2,3,4,5,6}

- <u>Subset (⊆):</u> B is subset of A means every elements of B is also an element of A and A = B or A ≠ B. Its is denoted by B ⊆ A
- <u>Proper Subset (⊂)</u>: B is proper subset of A means every element of B is an element of A

but $B \neq A$. It is denoted by $B \subset A$.

- <u>Disjoint Sets:</u> Disjoint sets do not have any element in common. If A and B are disjoint sets, then $A \cap B = \emptyset$
- <u>Intersection of Sets:</u> The intersection of sets A and B is the set of elements which are common to both A and B. It is denoted by $A \cap B$.
- <u>Union of Sets:</u> The union of sets A and B is set of elements which are in A, or in B, or in both A and B. It is denoted by $A \cup B$.
- <u>Complement of Set:</u> The complement of set A is a set which contain elements of universal set but not the elements of A. It is denoted by $\acute{A}$. $\acute{A} = \varepsilon - A$
- <u>Number of elements:</u> The number of elements of set A is denoted by n(A).
- $n(\emptyset)=0$ $n(\acute{A}) = n(\varepsilon) - n(A)$
- <u>for not disjoint sets</u> $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ and $n(A \cap B) \neq 0$
- <u>for disjoint sets</u> $n(A \cup B) = n(A) + n(B)$ and $n(A \cap B) = 0$

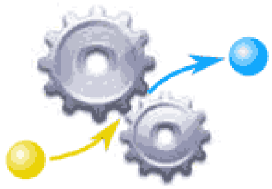
Unit-9

Functions

<u>Key Points</u>

What is a Function?

A function relates an input to an output. It is like a machine that has an input and an output and the output is related somehow to the input.



f(x)	f(x) is the classic way of writing a function. And there are other ways, as you will see!

<u>Input,</u>   <u>Relationship, Output</u>

We will see many ways to think about functions, but there are always three main parts:

- **The input**
- **The relationship**
- **The output**

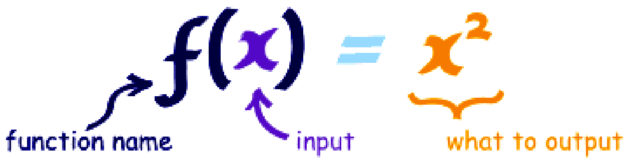
Here are the three parts:**Input**	*Relationship*	**Output**
0	× 2	0
1	× 2	2
7	× 2	14
10	× 2	20
...	...	...

<u>Example:</u> "Multiply by 2" is a very simple function.

<u>Examples</u> **of Functions:** x^2 (squaring) is a function, x^3+1 is also a function

Names of Function

First, it is useful to give a function a name. The most common name is "f", but we can have other names like "g" ... or even "marmalade" if we want. But let's use "f"



Example: $f(x) = x^2$:

- an input of 4
- becomes an output of 16.

In fact we can write **f(4) = 16**. The "x" is Just a Place-Holder!

Don't get too concerned about "x", it is just there to show us where the input goes and what happens to it. It could be anything!

Let function $f(x) = 1 - x + x^2$

Is the same function as:

- $f(q) = 1 - q + q^2$
- $h(A) = 1 - A + A^2$
- $w(\theta) = 1 - \theta + \theta^2$

The variable (x, q, A, etc) is just there so we know where to put the values: $f(\mathbf{2}) = 1 - \mathbf{2} + \mathbf{2}^2 = 3$

Sometimes there is No Function Name

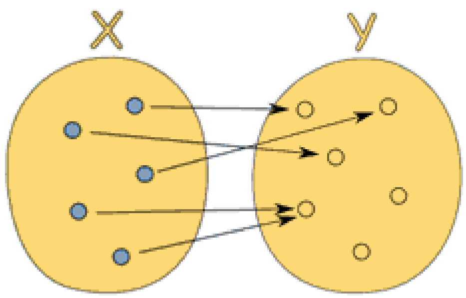
Sometimes a function has no name, and we see something like: $\mathbf{y = x^2}$

But there is still: an input (x), a relationship (squaring), and an output (y)

Function has special rules:

- It must work for **every** possible input value
- And it has only **one relationship** for each input value

This can be said in one definition:



Formal Definition of a Function

A function relates **each element** of a set with **exactly one** element of another set (possibly the same set).

The Two Important Things!

1. "...each element..." means that every element in **X** is related to some element in **Y**.

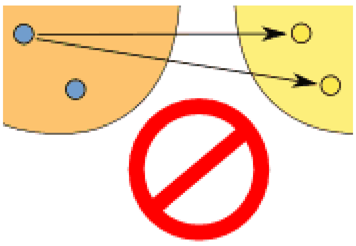
   We say that the function ***covers*** **X** (relates every element of it).

   (But some elements of **Y** might not be related to at all, which is fine.)

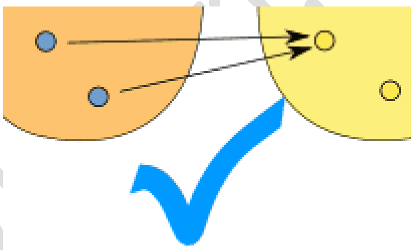
2. "...exactly one..." means that a function is ***single valued***. It will not give back 2 or more results for the same input.

So "f(2) = 7 **or** 9" is not right!

Note: "One-to-many" is **not** allowed, but "many-to-one" is allowed:

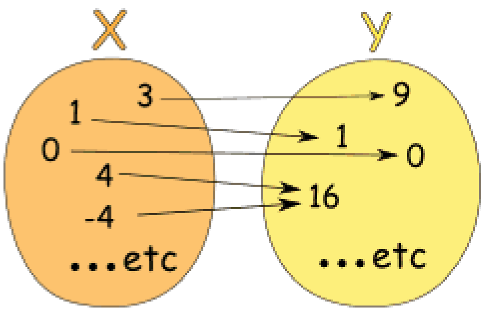


(one-to-many)
This is **NOT** OK in a function



(many-to-one)
But this **is** OK in a function

When a relationship does **not** follow those two rules then it is **not a function** ... it is still a **relationship**, just not a function.



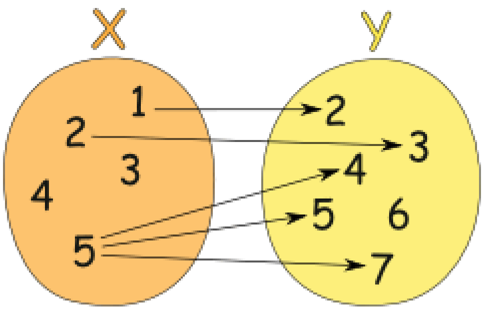
Example: The relationship $x \to x^2$

It is a function, because:

- Every element in X is related to Y
- No element in X has two or more relationships

So it follows the rules.

(Notice how both **4** and **-4** relate to **16**, which is allowed.)



Example: This relationship is not a function:

It is a **relationship**, but it is **not a function**, for these reasons:

- Value "3" in X has no relation in Y
- Value "4" in X has no relation in Y
- Value "5" is related to more than one value in Y

(But the fact that "6" in Y has no relationship does not matter)

Domain, Co-domain and Range

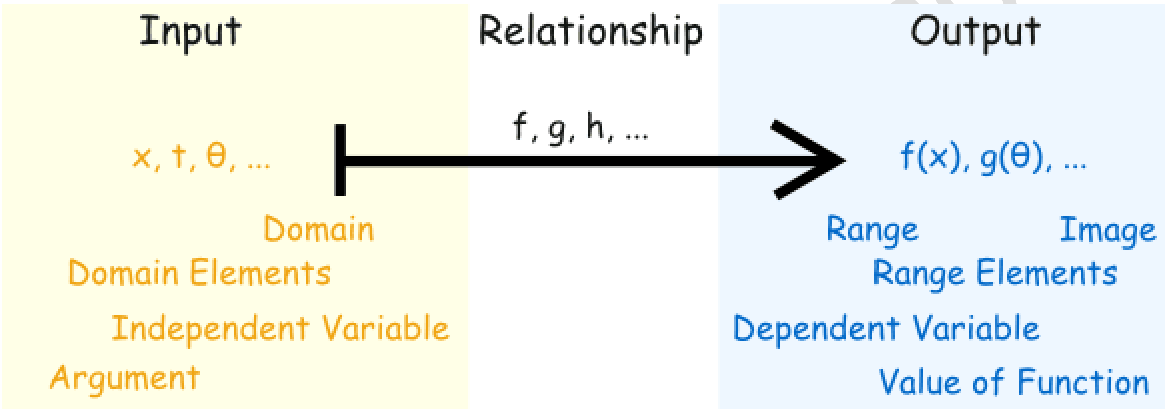
In above examples

- the set "X" is called the **Domain**,
- the set "Y" is called the **Co-domain**, and
- the set of elements that get pointed to in Y (the actual values produced by the function) is called the **Range.**

So Many Names!

Functions have been used in mathematics for a very long time, and lots of different names and ways of writing functions have come about.

Here are some common terms you should get familiar with:



Example: with $z = 2u^3$:

- "u" could be called the "independent variable"
- "z" could be called the "dependent variable" (it **depends on** the value of u)

Example: with **f(4) = 16**:

- "4" could be called the "argument"
- "16" could be called the "value of the function"

Composite Functions: (For IGCSE)

For two functions f and g, we the composite functions are

i) f g where $fg(x) = f(g(x))$

ii) g f where $gf(x) = g(f(x))$

Inverse of a Composite Function:

$(gf)^{-1} = f^{-1}g^{-1}$ holds for any two one-one functions

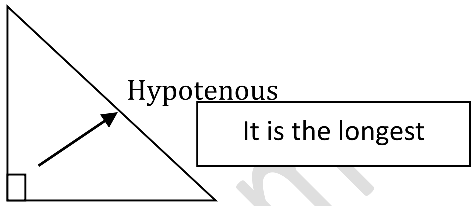
Unit-10

Trigonometry

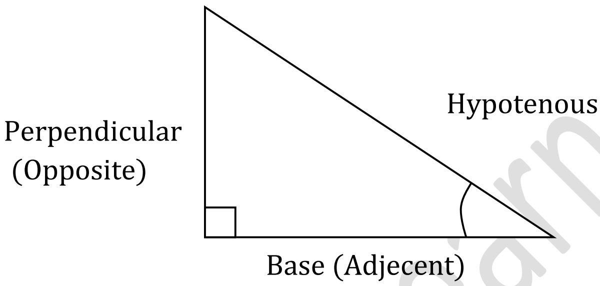
Key Points

Pythagoras' Theorem

$(\text{Hypotenous})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$



Trigonmetrical Ratios

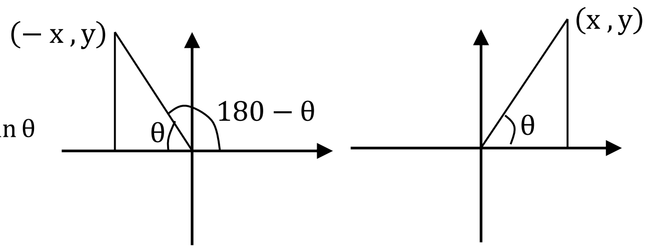


$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenous}} \qquad \cos\theta = \frac{\text{Base}}{\text{Hypotenous}} \qquad \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

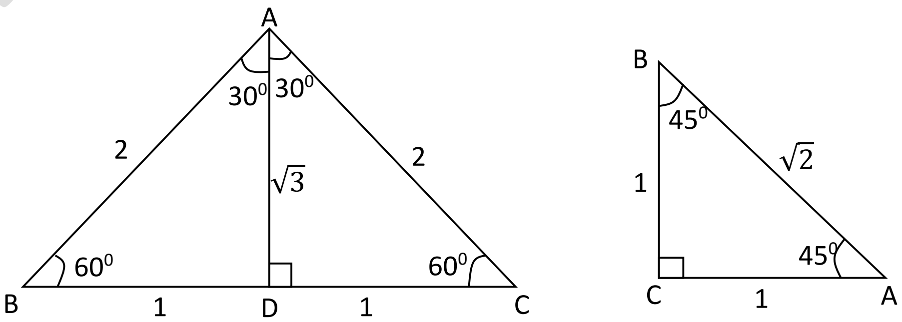
Trigonmetrical Ratios For Obtuse Angles

$\sin(180 - \theta) = \sin\theta$

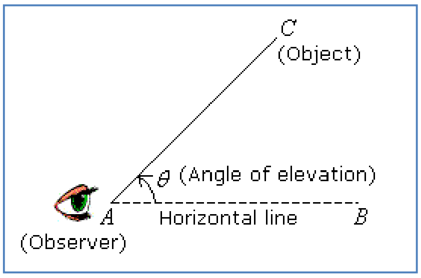
$\cos(180 - \theta) = -\cos\theta \qquad \tan(180 - \theta) = -\tan\theta$



Trigonmetrical Ratios of Special Angles: 30^0 , 60^0 , 45^0

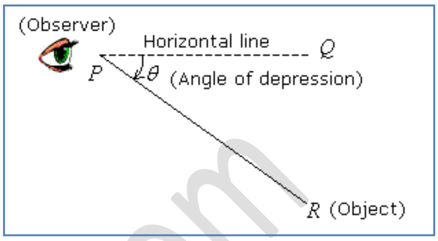


Angle of Elevation



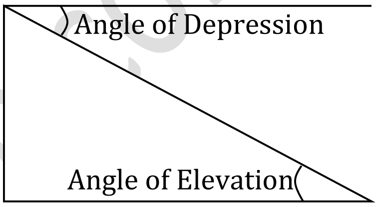
The Angle of Elevation is the angle between a horizontal line and the line joining the observer's eye to some object above the horizontal line.

Angle of Depression



The Angle of Depression is the angle between a horizontal line and the line joining the observer's eye to some object beneath the horizontal line.

Angle of Elevation = Angle of Depression



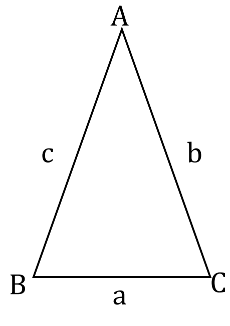
Greatest angle of elevation/depression:

Angle of elevation or depression is greatest when point of observation is nearest to the building.

Sine Rule

In any given triangle, the ratio of the length of a side and the sine of the angle opposite that side is a constant.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



The Sine Rule is used in the solution of triangles when at least either of the following is known:

a) Two angles and a side;
b) Two sides and an angle opposite a given side.

Cosine Rule

The Cosine Rule is used in the solution of triangles when two sides and an included angles are given OR when the three sides are given.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad b^2 = a^2 + c^2 - 2ac\cos B \qquad c^2 = a^2 + b^2 - 2ab\cos C$$

Area of Triangle

$$\text{Arae of } \Delta = \frac{1}{2} \times \text{base} \times \text{height} \qquad \text{Area of } \Delta = \frac{1}{2} ab\sin C$$

Unit-11

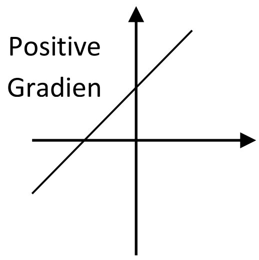
Coordinate Geometry

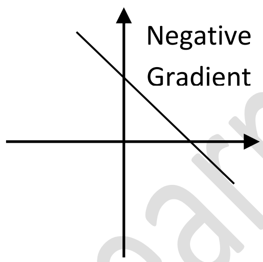
Key Points

- **Distance Formula**

The distance between two points P (-x_1, y_1)and Q ($x_2 y_2$) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

- **Gradient of Straight Line (m)**
  - Gradient $= \frac{\text{rise}}{\text{run}}$
  - The gradient of the line joining P(x_1, y_1)and Q(y_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.





- **Perpendicular Lines**
  - For perpendicular lines: $m_1 m_2 = -1$
  - If the gradient of a line is m then the gradient of its normal is $-\frac{1}{m}$

- **Parallel Lines**
  - Parallel lines have equal gradients. $m_1 = m_2$

- **Collinear Points**
  - Collinear points are points that lie on the same straight line.
  - Let there are three point A, B and C lie on the same straight line then gradient between point A and point B is equal to the gradient between point B and point C.

- **Mid Point formula** Mid-point between (x_1, y_1) & (x_2, y_2) is

$$\left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right]$$

- **Equation Of Straight Line:** $y = mx + c$

- The equation of a straight line which cuts off intercepts a and b on the x-axis and y-axis is

$$\frac{x}{a} + \frac{y}{b} = 1$$

- Equation of a straight line parallel to the y-axis at a distance 'a' from it is x=a.
- Equation of a straight line parallel to the x-axis at a distance 'b' from it is y=b.
- Equation of x-axis is y=0
- Equation of y-axis is x=0.
- The equation of a straight line passing through the origin (0,0) is y=m x.

➢ **Area of a triangle**

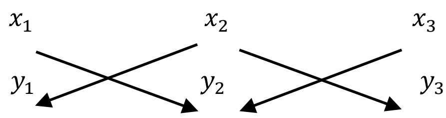
- Draw the triangle by using the given coordinates and calculate the area of triangle by applying the formula: area of triangle $= \frac{1}{2} \times h \times b$,
  where h= height of triangle of triangle and b= base of triangle.

OR

- The area of the triangle formed by the three points (x_1, y_1), (x_2, y_2), (x_3, y_3) is given by

$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

OR



$$\text{Area} = \frac{x_1y_2 + x_2y_3 + x_3y_1 - x_3y_2 - x_2y_1 - x_1y_3}{2}$$

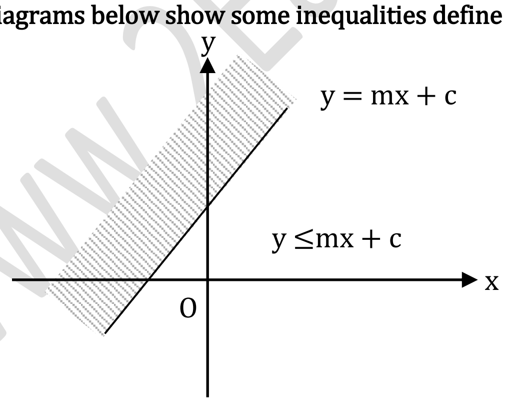
Unit-12

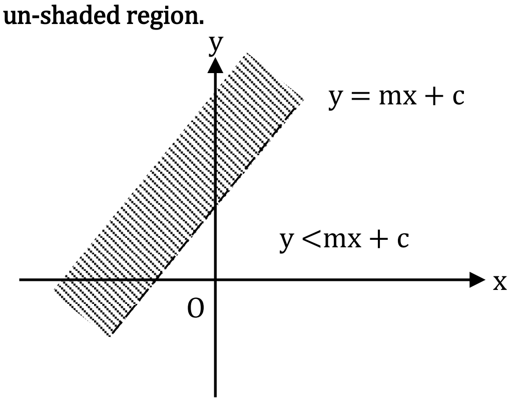
Inequalities

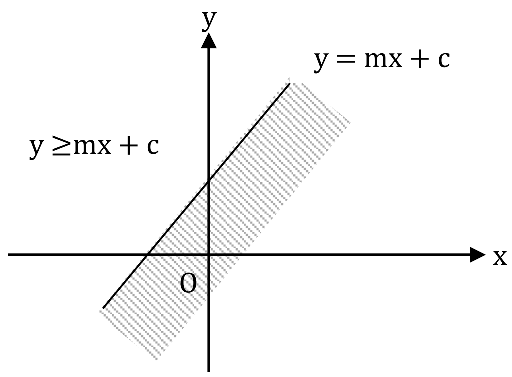
Key Points

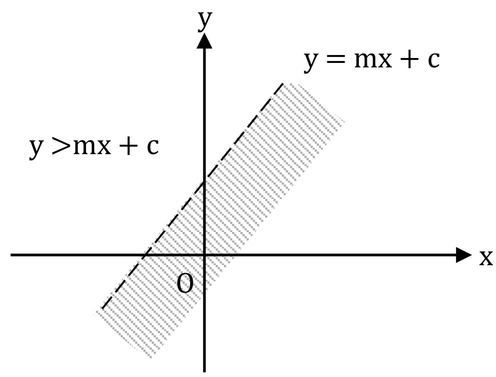
Graphical Representation of Linear Inequalities

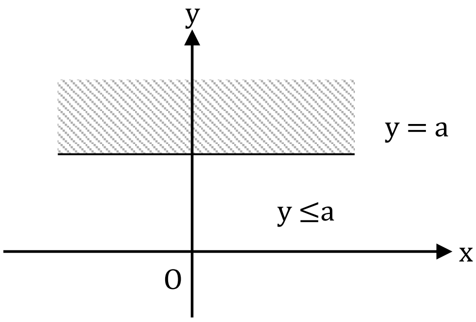
➢ **The diagrams below show some inequalities define the un-shaded region.**

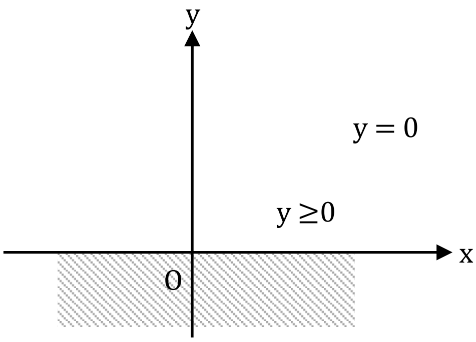








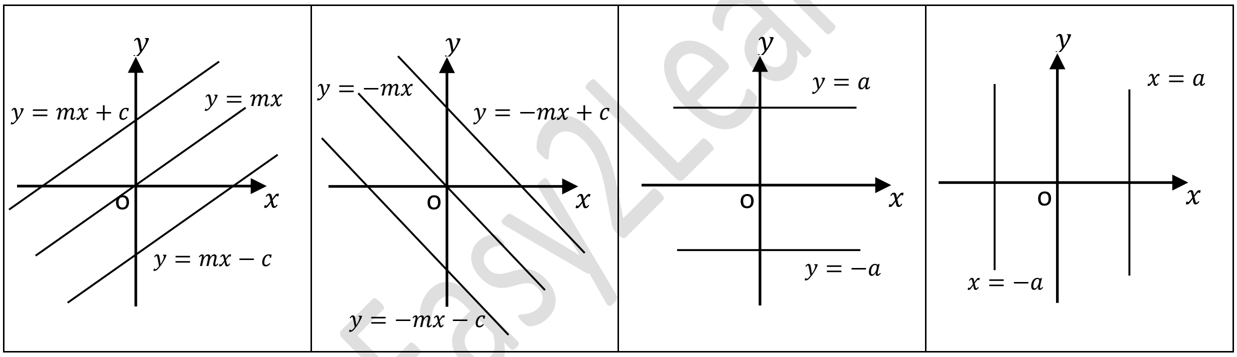




Unit-13

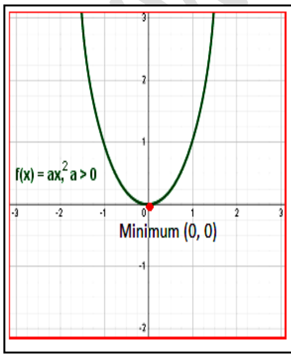
Graphs of Functions and Graphical Solutions

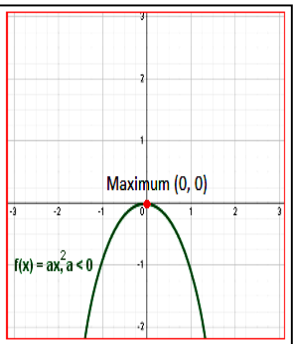
<u>Graphs of Linear Function:</u> $f(x) = m x + c$

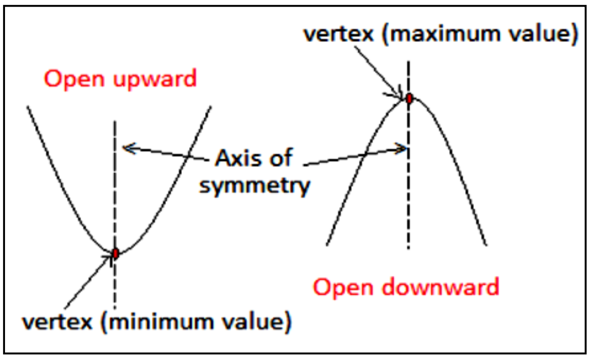


<u>Graphs of Quadratic Function :</u> $f(x) = ax^2 + b x + c$

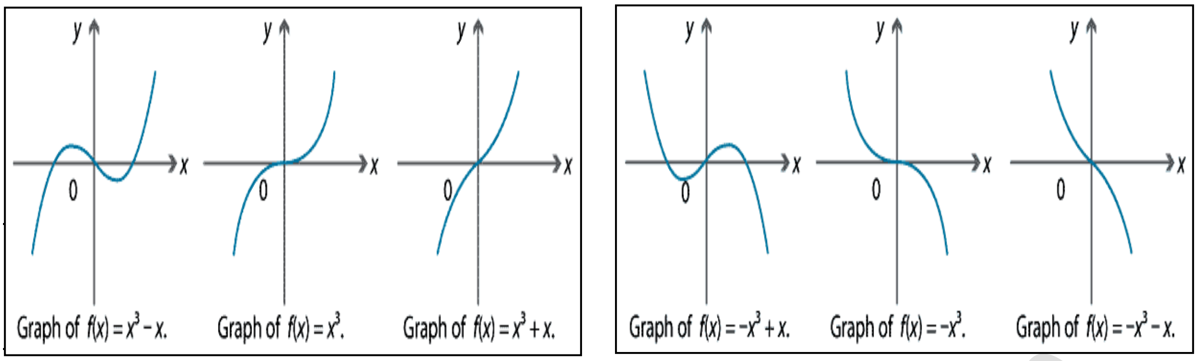
Graphing Quadratic Functions in Vertex Form : A quadratic function $f(x) = ax^2 + b x + c$ can be expressed in vertex form as $y = a(x - h)^2 + k$, where the vertex of the graph is **(h, k)** and the axis of symmetry is **x = h.**







Graphs of Cubic Functions: $f(x) = a\,x^3 + b\,x^2 + c\,x + d$

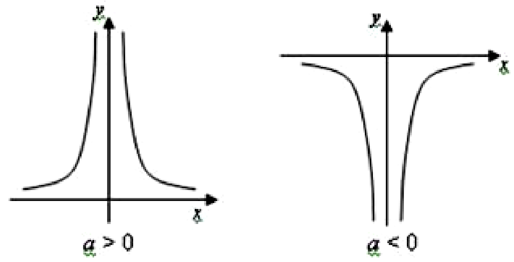


Graphs of Reciprocal Functions:

$n = -2$

$y = ax^{-2} \Rightarrow y = \dfrac{a}{x^2}$

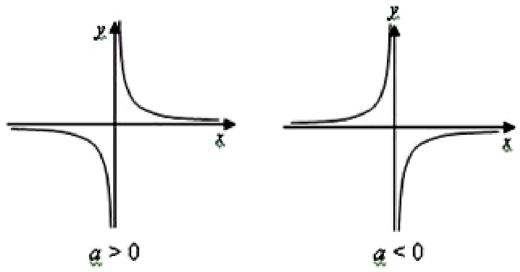
$\boxed{x^{-2} = \dfrac{1}{x^2}}$



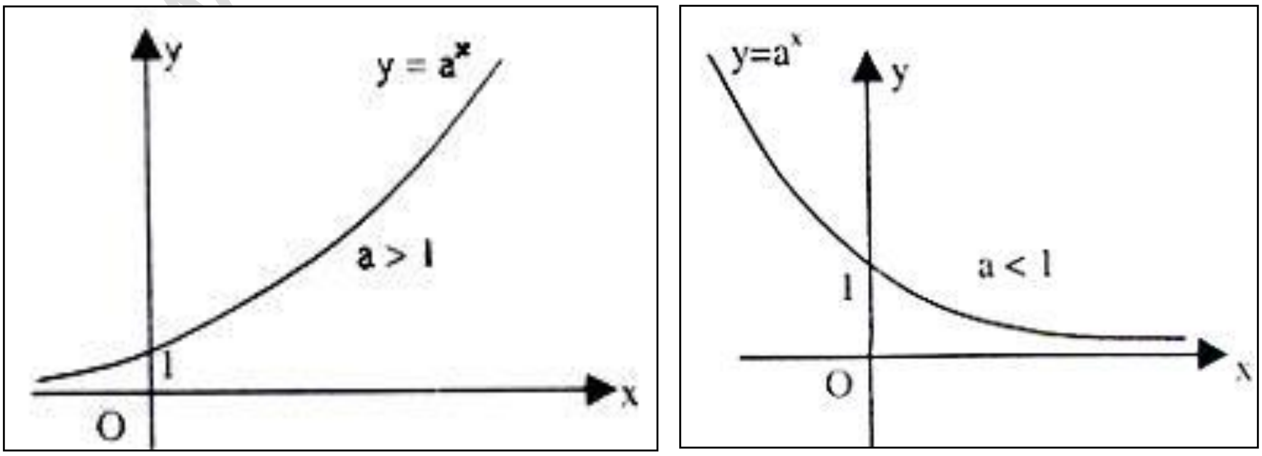
$n = -1$

$y = ax^{-1} \Rightarrow y = \dfrac{a}{x}$

$\boxed{x^{-1} = \dfrac{1}{x}}$



Graphs of Exponential Functions



Unit-14

Graphs in Practical Situation and Travel Graphs

Key Points

Speed: Speed can be defined as the distance covered by a moving object in unit time taken. SI unit of speed is m/s or ms^{-1}. **Speed is a scalar quantity.**

Uniform Speed: An object is said to be moving with uniform speed if it covers equal distances in equal intervals of time.

Variable Speed: An object is said to be moving with variable speed or non-uniform speed if it covers equal distances in unequal intervals of time or vice-versa.

Instantaneous Speed: The speed that the body possesses at a particular instant of time, is called instantaneous speed.

Average Speed: When we travel in a vehicle the speed of the vehicle changes from time to time depending upon the conditions existing on the road. In such a situation, the speed is calculated by taking the ratio of the total distance traveled by the vehicle to the total time taken for the journey. This is called the average speed

$$\text{Average Speed} = \frac{\text{Total Distance Traveled}}{\text{Total Time Taken}}$$

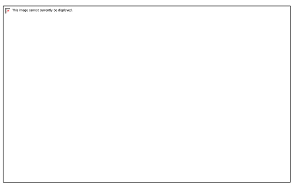
Acceleration: Acceleration is defined as the rate of change of velocity of a moving body with time. This change could be a change in the speed of the object or its direction of motion or both. Let an object moving with an initial velocity 'u' attain a final velocity 'v' in time 't', then acceleration 'a' produced in the object is

Acceleration = Rate of change of velocity with time

$$Acceleration = \frac{Change\ in\ velocity}{Time} \qquad a = \frac{v-u}{t}$$

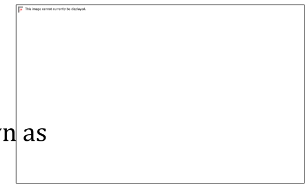
The SI unit of velocity is m/s and time is s

$\therefore$ **SI unit of acceleration is** $\frac{\frac{m}{s}}{s} = \frac{m}{s^2}$ **Acceleration is a vector quantity.**



Positive Acceleration : If the velocity of an object increases then the object is said to be moving with positive acceleration.

Example: A ball rolling down on an inclined plane.

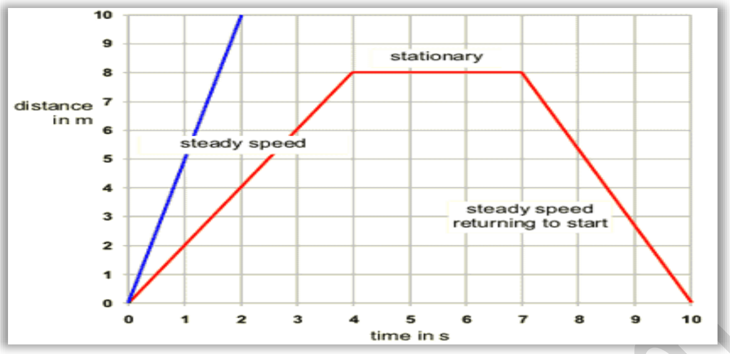


Negative Acceleration : If the velocity of an object decreases then the object is said to be moving with negative acceleration. Negative acceleration is also known as retardation or deceleration.

Examples: A ball moving up an inclined plane or a ball thrown vertically upwards is moving with a negative acceleration as the velocity decreases with time.

Zero Acceleration: If the change in velocity is zero, i.e., either the object is at rest or moving with uniform velocity, then the object is said to have zero acceleration.

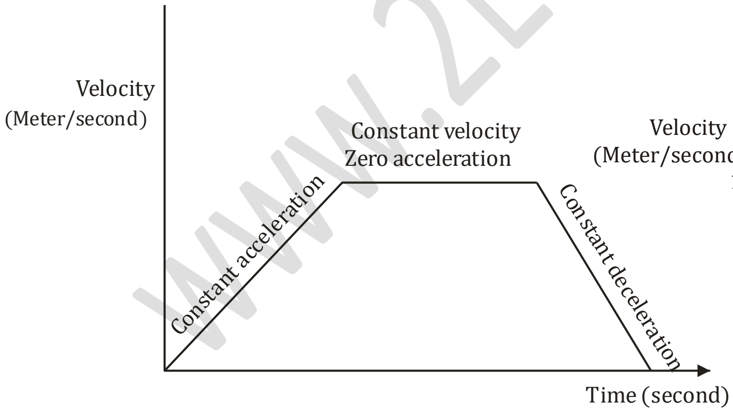
The Distance-Time Graph

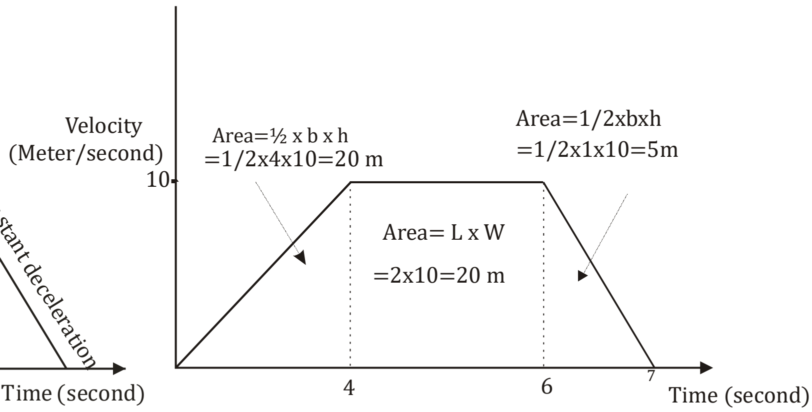


- Vertical coordinate shows distance.
- Horizontal coordinates shows time.
- Slope (gradient) shows speed.
- Straight line segments indicate constant speed.
- Graph getting steeper indicates getting faster, graph getting shallower indicates slowing down
- Level parts (horizontal line) indicate stopping

Speed-time graphs

- Vertical coordinate shows Speed.
- Slope (gradient) shows constant acceleration.
- Horizontal line segments indicate constant speed (acceleration is zero)..
- Area under the curve shows distance.
- Moving away from the horizontal axis indicates getting faster, moving towards the horizontal axis indicates getting slower.





- Graph above the horizontal axis indicates moving forward, graph below the horizontal axis indicates moving backward.
- Points on the horizontal axis indicate stopping.

Unit-15

Statistics

Key Point

- **Statistics**
  Statistics is a branch of mathematics dealing with the collection, analysis, interpretation, presentation, and organization of data.

- **Data**
  Data is a collection of facts, such as numbers, words, measurements, observations or even just descriptions of things.

- **Types of data**
  Data can be qualitative or quantitative.

  - Qualitative data is descriptive information (it *describes* something)

  - Quantitative data is numerical information (numbers). Quantitative data can also be Discrete or Continuous.
    - **Discrete data** can only take certain values (like whole numbers). Discrete data is counted.
    - **Continuous data** can take any value (within a range). Continuous data is measured

Example: What do you know about cat?

Qualitative:
- It is brown and black
- It has short hair
- it has lots of energy

Quantitative:
- Discrete:
  - It has 4 legs
  - It has 2 brothers
- Continuous:
  - It weighs 3.5 kg
  - It is 250 mm tall

More Examples:

Qualitative:
- Your friends' favorite holiday destination
- The most common given names in your town
- How people describe the smell of a new perfume

Quantitative:
- Height (Continuous)
- Weight (Continuous)
- Petals on a flower (Discrete)
- Customers in a shop (Discrete)

- **Collection of Data**
  Data can be collected in many ways. The simplest way is direct observation or by doing survey.

Example:
You want to find how many cars pass by a certain point on a road in a 10-minute interval.
So: stand at that point on the road, and count the cars that pass by in that interval.

Frequency
Frequency is how often something occurs.
Example: Sam played football on, Saturday Morning,, Saturday Afternoon, Thursday Afternoon
The frequency was 2 on Saturday, 1 on Thursday and 3 for the whole week.

Frequency Distribution
By counting frequencies we can make a Frequency Distribution table.

Example: These are the numbers of newspapers sold at a local shop over the last 10 days:

22, 20, 18, 23, 20, 25, 22, 20, 18, 20

Let us count how many of each number there is:

Ungrouped Data

Papers Sold	Frequency (days)
18	2
19	0
20	4
21	0
22	2
23	1
24	0
25	1

Grouped Data

It is also possible to **group** the values. Here they are grouped in 5s:

Papers Sold	Frequency (days)
15-19	2
20-24	7
25-29	1

- **Measures of Central Tendency**
  A measure of central tendency is a single value that describes the way in which a group of data cluster around a central value. To put in other words, it is a way to describe the center of a data set. There are three measures of central tendency: the mean, the median, and the mode.

- **Importance of Central Tendency**
  - It lets us know what is normal or 'average' for a set of data.
  - It also condenses the data set down to one representative value, which is useful when you are working with large amounts of data.
  - Central tendency also allows you to compare one data set to another.
  - Central tendency is also useful when you want to compare one piece of data to the entire data set.

- **Types of Central Tendency**
  There are three measures of central tendency: the mean, the median, and the mode.

- **Mean**
  The arithmetic mean is the most common measure of central tendency. It is simply the sum of the numbers divided by the number of numbers.

  - **Mean form Ungrouped Data**

$$\text{Mean} = \frac{\text{sum of terms}}{\text{number of terms}}, \quad \text{sum of terms} = \text{mean} \times \text{number of terms}, \quad \overline{X} = \frac{\sum fx}{\sum f}$$

  - **Mean from Grouped Data (Estimated Mean)**

  Example:

Time(Seconds)	Frequency
51 - 55	2
56 - 60	7
61 - 65	8
66 - 70	4

Mean can be estimated by using the **midpoints**. The groups (51-55, 56-60, etc), also called class intervals, are of width 5. The midpoints are in the middle of each class: 53, 58, 63 and 68.

Midpoint x	Frequency f	Midpoint × Frequency fx
53	2	106
58	7	406
63	8	504
68	4	272
Totals:	**21**	**1288**

$$\text{Estimated Mean} = \frac{\sum fx}{\sum f} = \frac{1288}{21} = 61.33$$

- **Median**
  The **median** is the value that cuts the data set in half. Median is a middle term of a set of data arrange in assending order.

  - **Median From Ungrouped Data**

  Median From List of Numbers

- If there are odd numbers of terms then there is one term in middle.

  Example1

  Given that the list of numbers 1, 1, 1, 1, 1, 13, 13, 13, 13, 13, 13

  Choose the middle number: 1, 1, 1, 1, 1, ***13***, 13, 13, 13, 13, 13

  The Median is **13**

- If there are even numbers of terms then there are two terms in middle.
  Average of twomiddle terms is middle.

Example 2

What is the Median of 3, 4, 7, 9, 12, 15

There are two numbers in the middle: 3, 4, ***7, 9***, 12, 15

So we average them: (7+9) / 2 = 16/2 = 8

The Median is 8

Median From Frequency Distribution Table

Example 1 : Find the median.

x	f	c.f
53	2	2
58	7	2+7=9
63	8	9+8=17
68	4	17+4=21
Totals:	**21**	

There are 21 terms. 11^{th} is middle term which is 63. So 63 is median.

Example 2 : Find the median.

x	f	c.f
53	5	5
58	7	5+7=12
63	6	12+6=18
68	6	18+6=24
Totals:	**24**	

There are 24 term therefore two terms will be in middle, 12^{th} and 13^{th}.

12^{th} term is 58 and 13^{th} term is 63. The median will be average of 58 and 63.

$$Median = \frac{58 + 63}{2} = \frac{121}{2} = 60.5$$

- **Mode**

  The most repeated term (highest frequency) is called mode.

  - **Mode from Ungrouped Data**

Example 1

Given that the list of numbers 1, 1, 1, 1, 1, 13, 13, 13, 13, 13, 13

"13" occurs 6 times, "1" occurs only 5 times, so the mode is **13**.

Example 2

What is the Mode of 3, 4, 4, 5, 6, 6, 7

4 occurs twice but 6 **also** occurs twice. So **both 4 and 6** are modes.

- **Mode from Grouped Data**

  Modal Class: The class with highest frequency for a group of data.
  Example

Time(Seconds)	Frequency
51 - 55	2
56 - 60	7
61 - 65	8
66 - 70	4

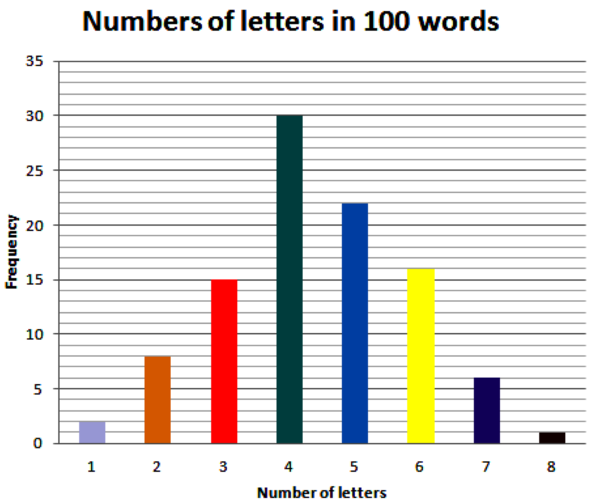
The modal group is **61 – 65** (the group with the highest frequency)

➢ **Measures of Spread**
The measures of spread are **Range, Quartiles, Interquartile Range and Percentile.**

➢ **The Range**
The Range is the difference between the lowest and highest values.

Example 1
In **{4, 6, 9, 3, 7}** the lowest value is 3, and the highest is 9.
So the **range** = 9 − 3 = **6.**

Example 2
Olivia wrote a 100 word story. She then counted the number of letters in each word. Her results are shown in the bar graph. What is the range?

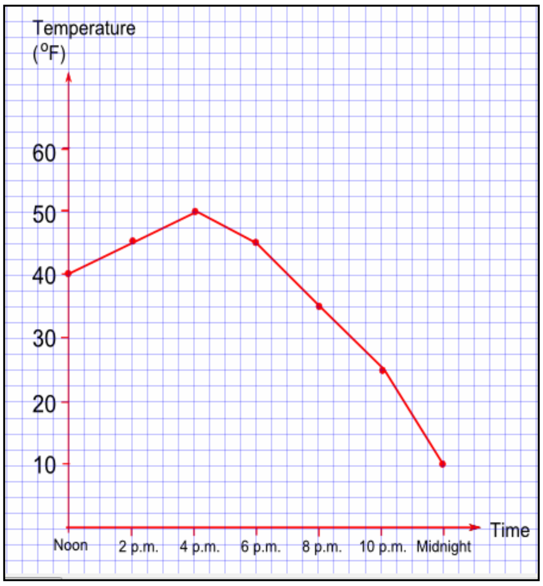


Solution:
The range is the difference between the lowest and highest values.

The highest value is the length of the longest word = 8
The lowest value is the length of the shortest word = 1

Therefore the **range** = 8 - 1 = 7

Example 3
Jerry recorded the temperature in his room every two hours over a 12 hour period from noon to midnight. The results are shown in the line graph.
What is the range?



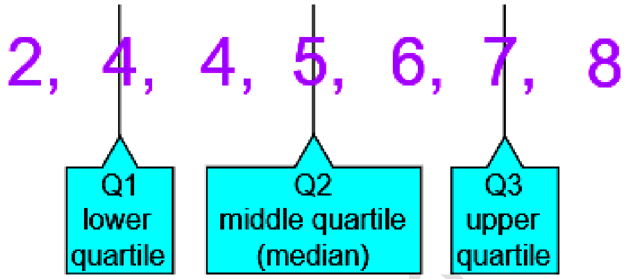
Solution:
The range is the difference between the lowest and highest values.

The highest value was 50°F
The lowest value was 10°F
Therefore the range = 50°F - 10°F = 40°F

- **Quartiles**

  Quartiles are the values that divide a list of numbers into quarters.
  Put the list of numbers in order, then cut the list into four equal parts.
  The Quartiles are at the "cuts".

  Example 1
  Given the list of numbers 5, 7, 4, 4, 6, 2, 8

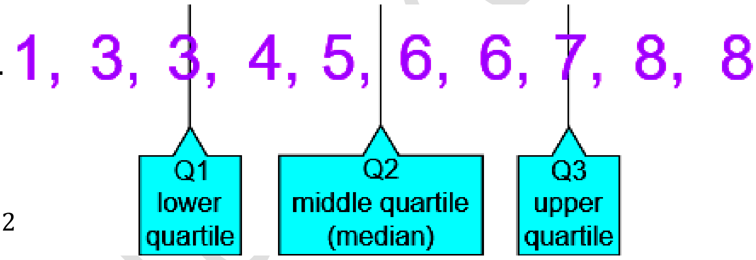


  Solution:
  Put them in order: 2, 4, 4, 5, 6, 7, 8
  Cut the list into quarters:

  Quartile 1 (Q1) = 4
  Quartile 2 (Q2), which is also the Median, = 5
  Quartile 3 (Q3) = 7

  Example 2
  Given the list of numbers 1, 3, 3, 4, 5, 6, 6, 7, 8, 8

  Solution:
  The numbers are already in order.
  Cut the list into quarters.
  In this case Quartile 2 is half way between 5 and 6:
  Q2 = (5+6)/2 = 5.5
  Quartile 1 (Q1) = 3 Quartile 2



- **Interquartile Range**

  The "Interquartile Range" is from Q1 to Q3.
  To calculate it just subtract Quartile 1 from Quartile 3

  Interquartile Range from **Example 1** $Q3 - Q1 = 7 - 4 = 3$

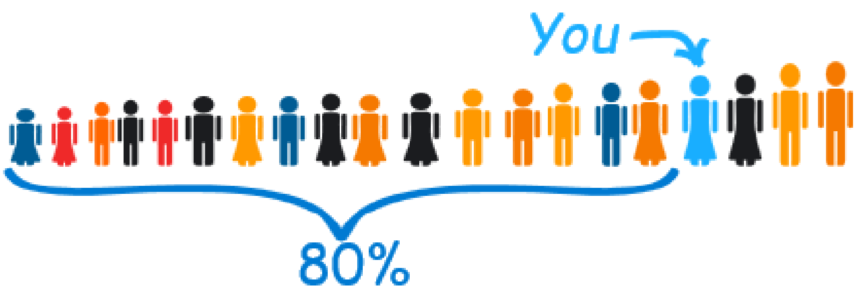
  Interquartile Range from **Example 2** $Q3 - Q1 = 7 - 3 = 4$

- **Percentiles**

  The value below which a percentage of data falls. To calculate the percentile the data must be in ascending order

  The **Quartiles** also divide the data into divisions of 25%, so:
  Quartile 1 (Q1) can be called the 25th percentile
  Quartile 2 (Q2) can be called the 50th percentile
  Quartile 3 (Q3) can be called the 75th percentile

  Example:
  You are the fourth tallest person in a group of 20. Find the percentile.
  Solution:
  80% of people are shorter than you.
  Its mean you are at the 80th percentile.



- **Pie Chart**

  A special chart that uses "pie slices" to show relative sizes of data. Data is represented by either in percentage or in the form of angle.

  Example:

  Imagine you survey your friends to find the kind of movie they like best.

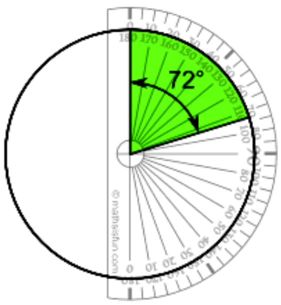
  - First, put your data into a table, then add up all the values to get a total.

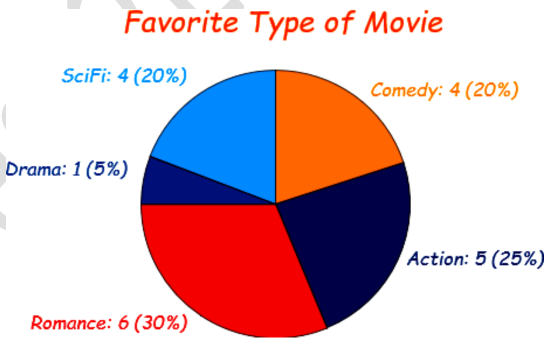
Favorite Type of Movie					Total
Comedy	Action	Romance	Drama	SciFi	
4	5	6	1	4	20

  - Divide each value by the total and multiply by 100 to get a percent.
  - To figure out how many degrees for each "pie slice" (correctly called a sector), divide each value by total and multiply by 360 (a full circle has 360 degrees) to get an angle.

Comedy	Action	Romance	Drama	SciFi	TOTAL
4	5	6	1	4	**20**
20%	**25%**	**30%**	**5%**	**20%**	**100%**
$4/20 \times 360° = $ **72°**	$5/20 \times 360° = $ **90°**	$6/20 \times 360° = $ **108°**	$1/20 \times 360° = $ **18°**	$4/20 \times 360° = $ **72°**	**360°**

Now draw a circle. Use protractor to measure the degrees of each sector. Label each sector by its name and angle/percentage.





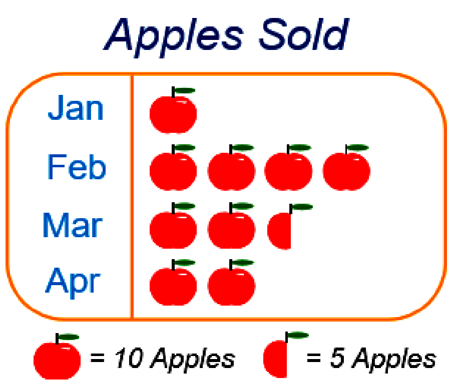
- **Pictographs**

  A Pictograph is a way of showing data using images. Each image stands for a certain number of things.

  Example

  Here is a pictograph of how many apples were sold at the local shop over 4 months.
  Note that each picture of an apple means **10 apples**
  (and the half-apple picture means 5 apples).
  So the pictograph is showing:

  - In January **10 apples** were sold
  - In February **40 apples** were sold
  - In March **25 apples** were sold
  - In April **20 apples** were sold



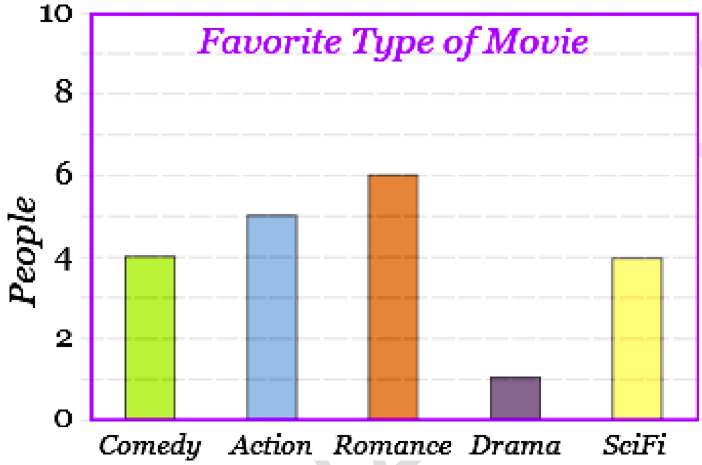
- **Bar Graphs**

A Bar Graph (also called Bar Chart) is a graphical display of data using bars of different heights.

Example

Imagine you just did a survey of your friends to find which kind of movie they liked best.

Favorite Type of Movie				
Comedy	Action	Romance	Drama	SciFi
4	5	6	1	4

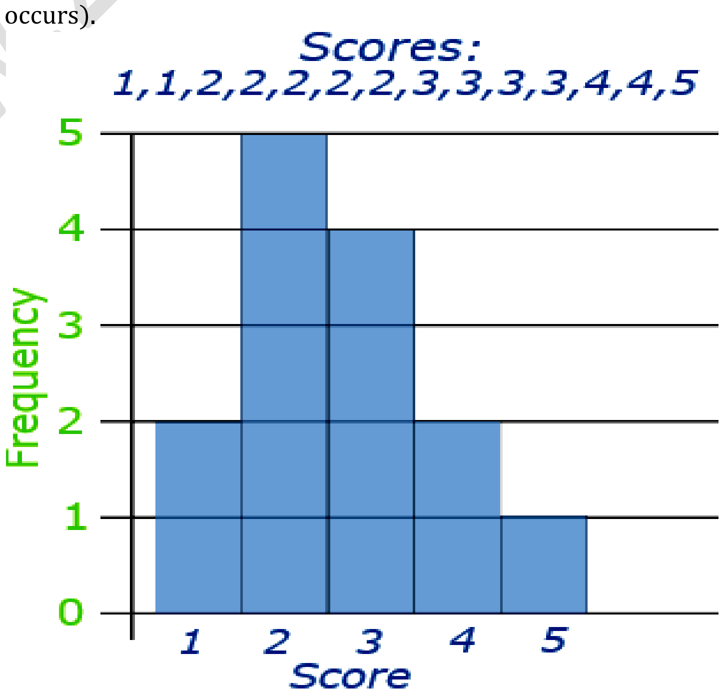


- **Histograms**

A graphical display of data using bars of different heights. Bars of histogram are joint with each other.

- **Histogram of Ungrouped Data**

A Frequency Histogram is a special histogram that uses vertical columns to show (how many times each score occurs).

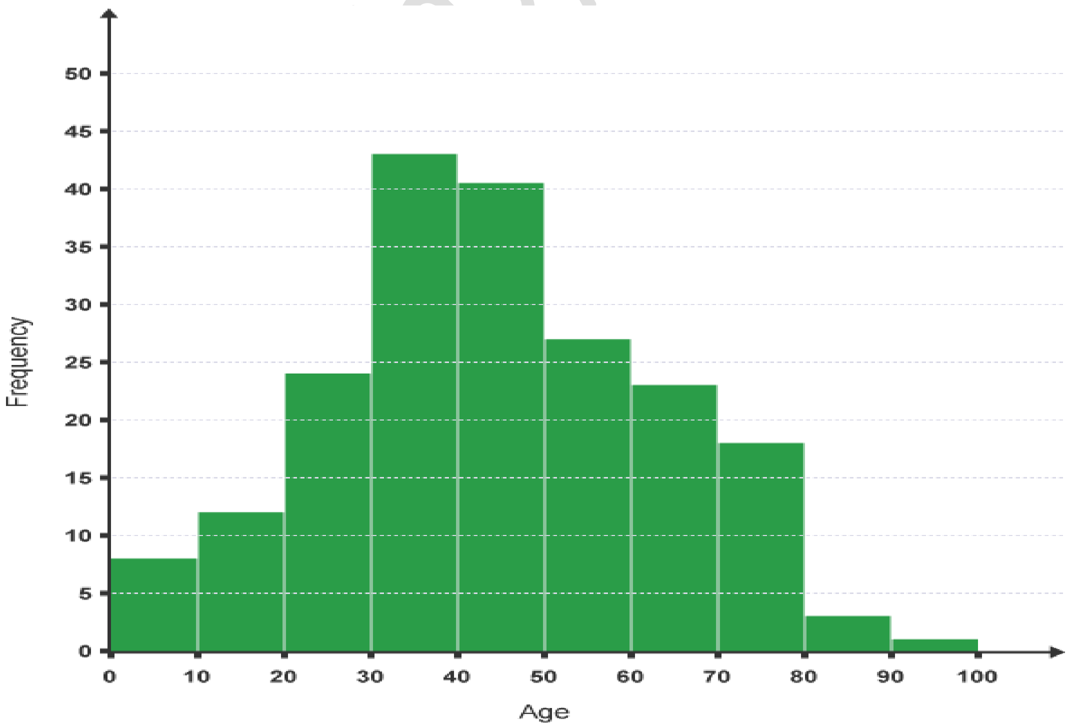


- **Histograms of Group Data with Equal Class Width**

When a large amount of data has to be collected, use a grouped frequency distribution. The following tally chart represents the ages of 200 people entering a park on a Saturday afternoon. The ages have been grouped into the classes 0-10, 10-20, 20-30, and so on. The class width of all groups is same.

To draw histogram, frequency is marked on vertical axis and age is marked on horizontal axis.

Age	Tally	Frequency
0 - 10	𝍸 |||	8
10 - 20	𝍸 𝍸 ||	12
20 - 30	𝍸 𝍸 𝍸 𝍸 ||||	24
30 - 40	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	43
40 - 50	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	41
50 - 60	𝍸 𝍸 𝍸 𝍸 𝍸 ||	27
60 - 70	𝍸 𝍸 𝍸 𝍸 |||	23
70 - 80	𝍸 𝍸 𝍸 |||	18
80 - 90	|||	3
90 - 100	|	1

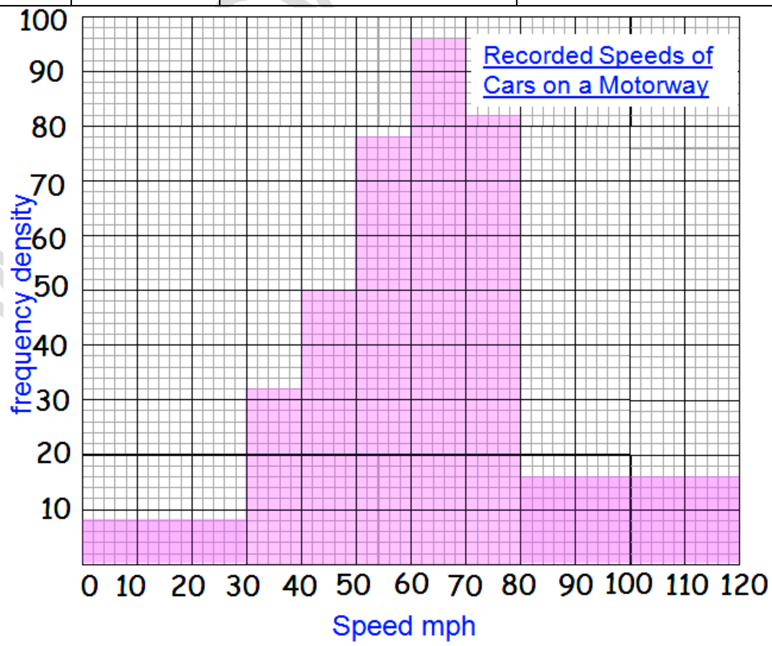


- **Histograms of Group Data with Unequal Class Width**

The frequency table gives information on the speeds (mph) of a sample of drivers using motorway. Construct a Histogram for this data.

Since class width of intervals is not equal therefore frequency density will be calculated.

Speed	freq	C.W	$f.d = \frac{f}{c.w}$
0 - 30	240	30 – 0 =30	$\frac{240}{30} = 8$
30 - 40	320	40 – 30 =10	$\frac{320}{10} = 32$
40 - 50	500	50 – 40 =10	$\frac{500}{10} = 50$
50 - 60	780	60 – 50 =10	$\frac{780}{10} = 78$
60 - 70	960	70 – 60 = 10	$\frac{960}{10} = 96$
70 - 80	820	80 – 70 = 10	$\frac{820}{10} = 82$
80 - 120	640	120 – 80=40	$\frac{640}{40} = 16$



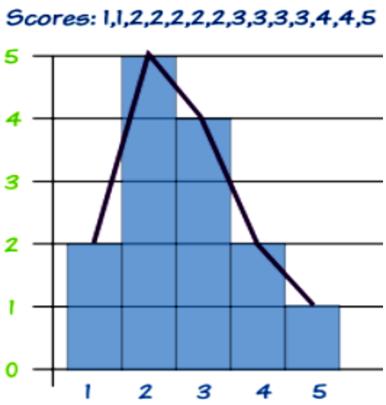
$$f.d = \frac{f}{c.w}$$

Area of rectangle = length x width

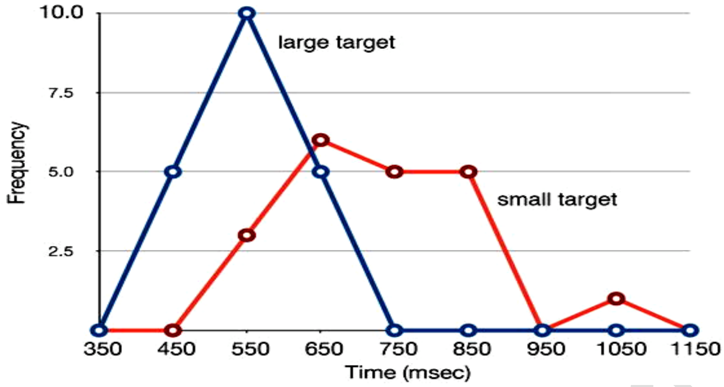
$$Frequency = f.d \times c.w$$

When constructing a histogram with non-uniform (unequal) class widths, we must ensure that the areas of the rectangles are proportional to the class frequencies.

- **Frequency Polygon**
  A graph made by joining the middle-top points of the columns of a frequency histogram

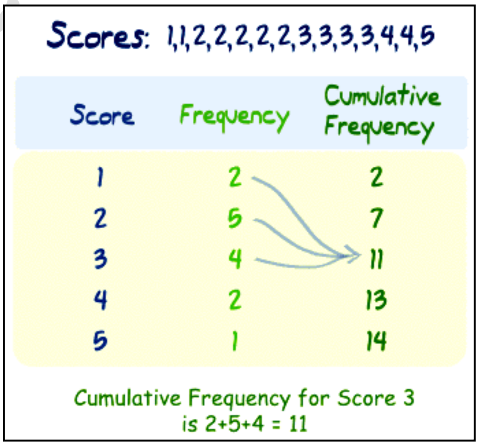


- **Overlaid frequency polygons**
  Frequency polygons are useful for comparing distributions.
  This is achieved by overlaying the frequency polygons drawn for different data sets.



- **Cumulative Tables and Graphs**

  Cumulative means "how much so far". Think of the word "accumulate" which means to gather together. To have cumulative totals, just add up the values as you go.



Scores: 1,1,2,2,2,2,2,3,3,3,3,4,4,5

Score	Frequency	Cumulative Frequency
1	2	2
2	5	7
3	4	11
4	2	13
5	1	14

Cumulative Frequency for Score 3 is 2+5+4 = 11

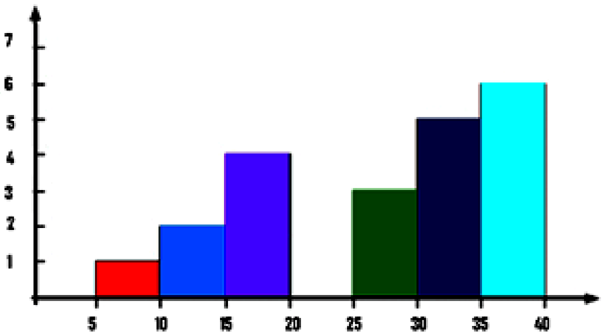
- **Cumulative Frequency**

  The total of a frequency and all frequencies so far in a frequency distribution. It is the 'running total' of frequencies.

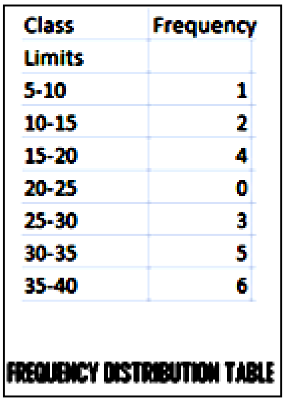
- **Cumulative Frequency Distribution Table**

  Example

  Draw the Cumulative frequency table by using given Histogram.



1. HISTOGRAM



Class Limits	Frequency
5-10	1
10-15	2
15-20	4
20-25	0
25-30	3
30-35	5
35-40	6

FREQUENCY DISTRIBUTION TABLE

Solution

Step 1 : Build a **frequency distribution table**, like the one to the right of the histogram above. Label column 1 with your class limits. In column 2, count the number of items in each class and fill the columns in as shown above. To fill in the columns, count how many items are in each class, using the histogram.

Step 2: Label a new column in your frequency distribution table "Cumulative frequency" and compute the it. The first entry will be the same as the first entry in the frequency column. The second entry will be the sum of the first two entries in the frequency column. The third entry will be the sum of the first three entries in the frequency column, the fourth will be the sum of the first four entries in the frequency column etc.

Class Limits	Frequency	Cumulative Frequency
5-10	1	1
10-15	2	3
15-20	4	7
20-25	0	7
25-30	3	10
30-35	5	15
35-40	6	21

- **Cumulative Frequency Curve**

A Cumulative Frequency Graph is a graph plotted from a cumulative frequency table. A cumulative frequency graph is also called an Ogive or cumulative frequency curve.

Example

Draw a cumulative frequency graph for the given frequency table.

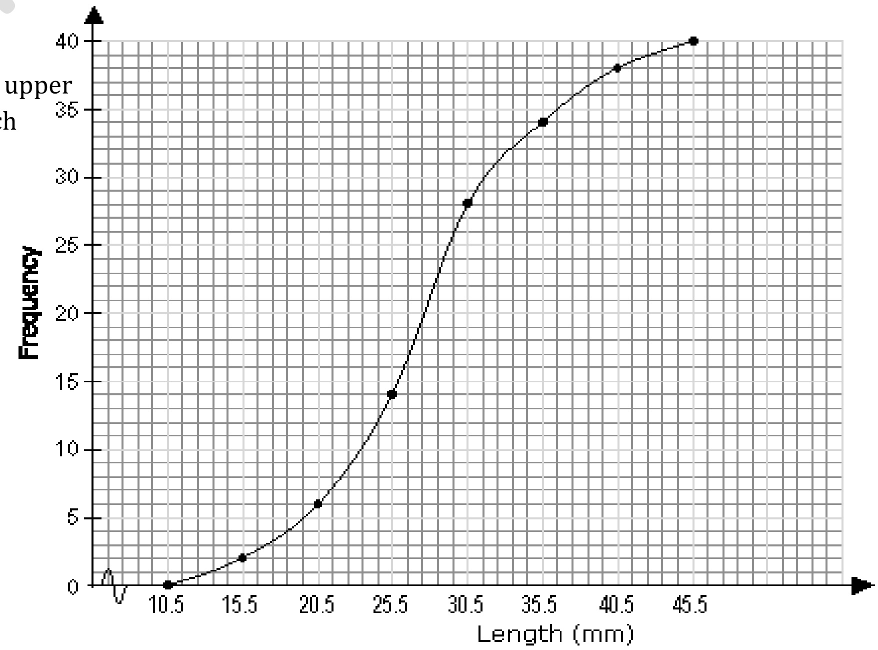
Length (x mm)	Frequency
11 – 15	2
16 – 20	4
21 – 25	8
25 – 30	14
31 – 35	6
36 – 40	4
41 – 45	2

Solution

We need to add a class with 0 frequency before the first class and then find the upper boundary for each class interval.

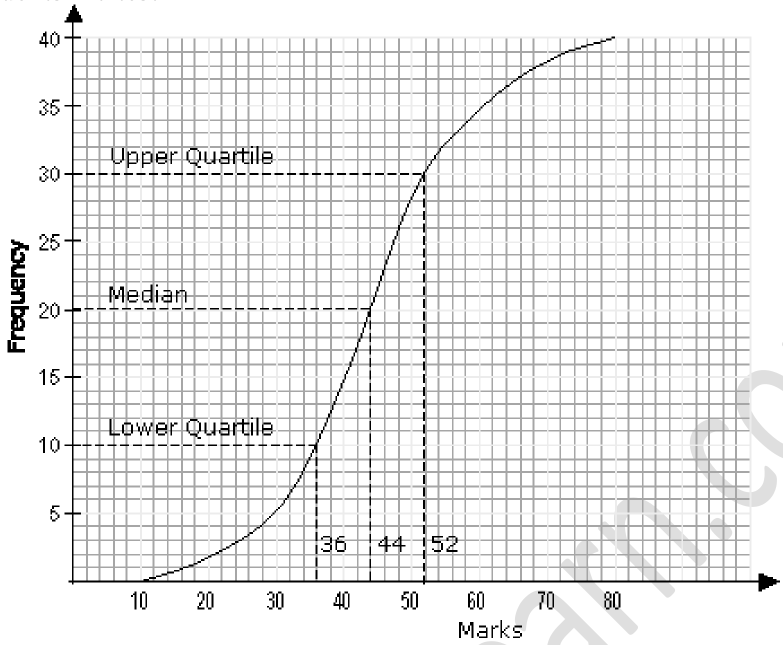
Length (x mm)	Frequency	Upper Class Boundary	Length (x mm)	Cumulative Frequency
6 – 10	0	10.5	$x \leq 10.5$	0
11 – 15	2	15.5	$x \leq 15.5$	2
16 – 20	4	20.5	$x \leq 20.5$	6
21 – 25	8	25.5	$x \leq 25.5$	14
25 – 30	14	30.5	$x \leq 30.5$	28
31 – 35	6	35.5	$x \leq 35.5$	34
36 – 40	4	40.5	$x \leq 40.5$	38
41 – 45	2	45.5	$x \leq 45.5$	40

Plot the cumulative frequency against the upper class boundary of each interval and join the points with a smooth curve.



- **Median, Quartiles And Percentiles from Cumulative frequency Curve**

The following cumulative frequency graph shows the distribution of marks scored by a class of 40 students in a test.



Use the graph to estimate
a) the median mark
b) the upper quartile
c) the lower quartile
d) the interquartile range

Solution:
a) **Median** corresponds to the **50th percentile** i.e. 50% of the total frequency.

50% of the total frequency = $\frac{50}{100} \times 40 = \frac{1}{2} \times 40 = 20$

From the graph, 20 on the vertical axis corresponds to 44 on the horizontal axis. The median mark is 44.

b) The **upper quartile** corresponds to the **75th percentile** i.e. 75% of the total frequency.

75% of the total frequency = $\frac{75}{100} \times 40 = \frac{3}{4} \times 40 = 30$

From the graph, 30 on the vertical axis corresponds to 52 on the horizontal axis. The upper quartile is 52.

c) The **lower quartile** corresponds to the **25th percentile** i.e. 25% of the total frequency.

25% of the total frequency = $\frac{25}{100} \times 40 = \frac{1}{4} \times 40 = 10$

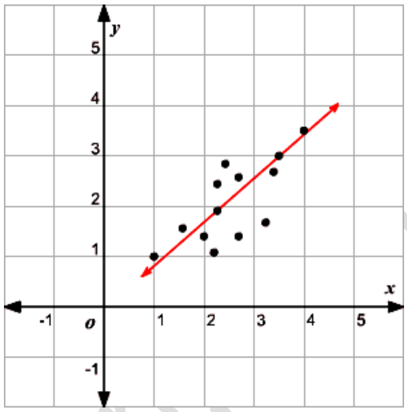
From the graph, 10 on the vertical axis corresponds to 36 on the horizontal axis. The lower quartile is 36.

d) The interquartile range = upper quartile – lower quartile = 52 – 36 = 16

➢ Scatter Diagram (Comparing Data)

A scatterplot (Scatter Diagram) is used to graphically represent the relationship between two variables. Explore the relationship between scatterplots and correlations, the different types of correlations, how to interpret scatterplots, and more.

Scatterplot has a horizontal axis (x-axis) and a vertical axis (y-axis). One variable is plotted on each axis.



- **Line of best fit**

  A line of best fit is a straight line drawn through the maximum number of points on a scatter plot balancing about an equal number of points above and below the line. It is used to study the nature of relation between two variables.

- **Correlation:**

While studying statistics, one comes across the concept of **correlation**. It is a statistical method which enables the researcher to find whether two variables are related and to what extent they are related. We can observe this when a change in one particular variable is accompanied by changes in other variables as well, and this happens either in the same or opposite direction, then the resultant variables are said to be correlated.

- **Properties of Correlation:**

All correlations have two properties: strength and direction.

The **strength of a correlation** is determined by its numerical value.

The strength of a correlation indicates how strong the relationship is between the two variables. The strength is determined by the numerical value of the correlation. A correlation of 1, whether it is +1 or -1, is a perfect correlation. In perfect correlations, the data points lie directly on the line of fit. The further the data are from the line of fit, the weaker the correlation. A correlation of *0* indicates that there is no correlation. The following should be considered when determining the strength of a correlation:

- The closer a positive correlation lies to +1, the stronger it is.
  - i.e., a correlation of +.87 is stronger than a correlation of +0.42.
- The closer a negative correlation is to -1, the stronger it is.
  - i.e., a correlation of -.84 is stronger than a correlation of -0.31.

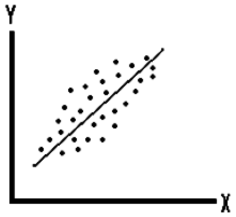
- When **comparing a positive correlation to a negative correlation**, only look at the numerical value. Do not consider whether or not the correlation is positive or negative. The correlation with the highest numerical value is the strongest.
  - i.e., a correlation of -.80 is stronger than a correlation of +.55.
- If the numerical values of a correlation are the same, then they have the same strength no matter if the correlation is positive or negative.
  - i.e., a correlation of -.80 has the same strength as a correlation of +.80.

The <u>direction of the correlation</u> is determined by whether the correlation is positive or negative.

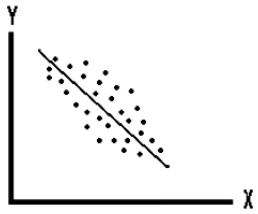
- <u>**Types of Correlation:**</u>

Positive correlation: Both variables move in the same direction. In other words, as one variable increases, the other variable also increases. As one variable decreases, the other variable also decreases, i.e. years of education and yearly salary are positively correlated.

Negative correlation: The variables move in opposite directions. As one variable increases, the other variable decreases. As one variable decreases, the other variable increases, i.e. hours spent sleeping and hours spent awake are negatively correlated.



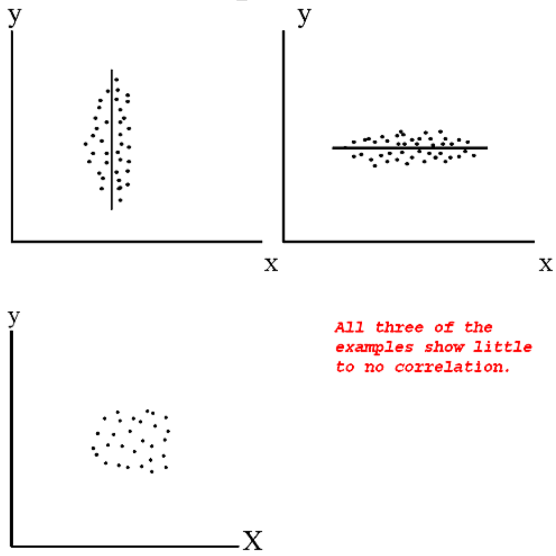
Positive Correlation



Negative Correlation

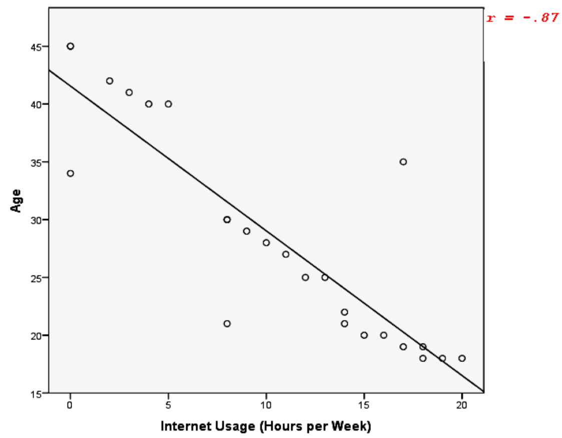
Zero Correlation (No Correlation)

What does it mean to say that two variables have no correlation? It means that there is no apparent relationship between the two variables. For example, there is no correlation between shoe size and salary. This means that high scores on shoe size are just as likely to occur with high scores on salary as they are with low scores on salary.



- <u>**Interpretations of Scatterplots**</u>

So what can we learn from scatterplots? Let's create scatterplots using some of the variables in our table. Let's first compare age to Internet use. Now let's put this on a scatterplot. Age is plotted on the y-axis of the scatterplot and Internet usage is plotted on the x-axis.We see that there is a negative correlation between age and Internet usage. That means that as age increases, the amount of time spent on the Internet declines, and vice versa. The direction of the scatterplot is a negative correlation. In the upper right corner of the scatterplot, we see r = -.87. Since r signifies the correlation, this means that our correlation is -.87 and it is strong correlation.



Unit-16

Symmetry

__Key Points__

Symmetry

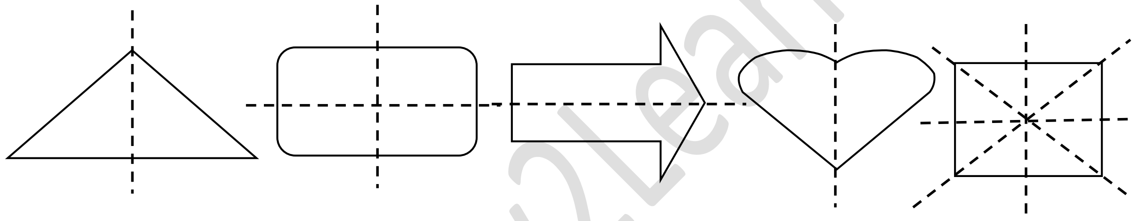
Symmetry is when one shape becomes exactly like another if you flip, slide or turn it. There are two types of symmetry, reflective symmetry and rotational symmetry.

Reflective symmetry:

An object has reflective symmetry if it can be reflected in a particular line and looks the same as the original.

Line of Symmetry:

Line of symmetry is a line in the figures that divide figures in two parts which are reflection of each other.

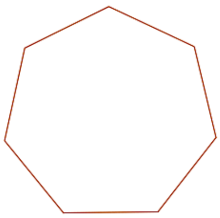


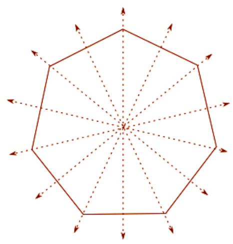
Number of Line of Symmetry:

- Its number of lines that divide the figures in two parts which are reflection of each other.
- For example letter H ,O and rectangular shape has two lines of symmetry, squared shape has four lines of symmetry.

Example

The diagram shows the outline of a British 50p coin.
How many lines of symmetry does it have?



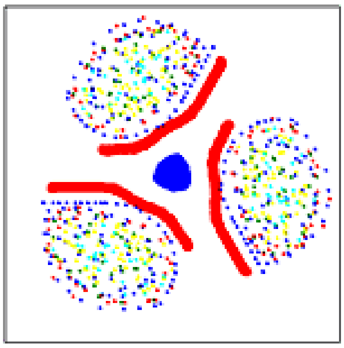


7 Lines of Symmetry

Solution: 7 lines of symmetry.

Explanation
It is a regular heptagon (7 sided figure) and has 7 lines of symmetry

Rotational Symmetry



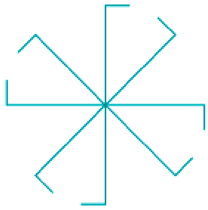
- If a figure is rotated about a point through an angel (other than 360^0) without changing its original position, it is said that the figure has a rotational symmetry about that point.

Order of Symmetry

- The order of rotational symmetry of a figures is the number of times the figure can be rotated without changing its original position.
- A figure with order of rotational symmetry 1 is said to have no rotational symmetry.

Example

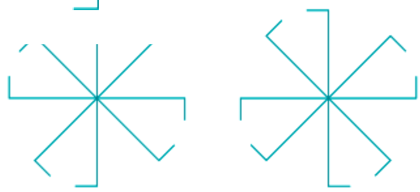
What is the order of rotational symmetry of this shape?



Solution: Order of rotational symmetry is 4.

Explanation

It does not look the same after a rotation of 45°. It requires a rotation of 90° before it looks the same.
So it looks the same after rotations of 90°, 180°, 270° and 360° takes it back to its original position. So the order of rotational symmetry is 4.

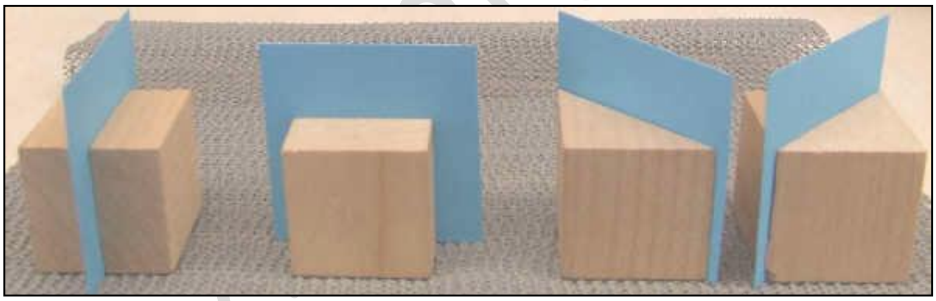


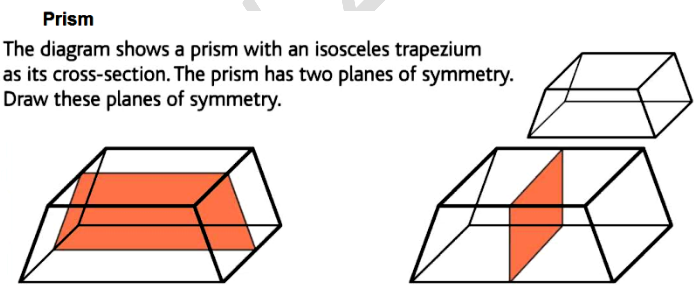
After a 45° rotation

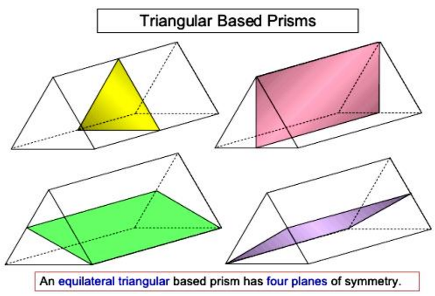
Plane of Symmetry

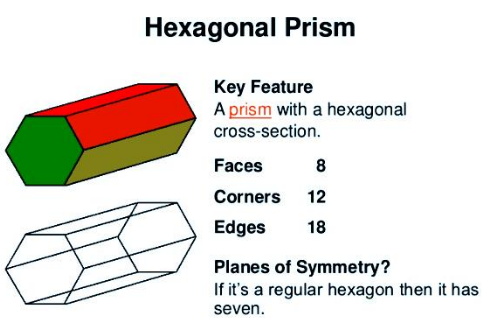
In mathematics, a plane is a flat, two-dimensional surface that extends infinitely far.

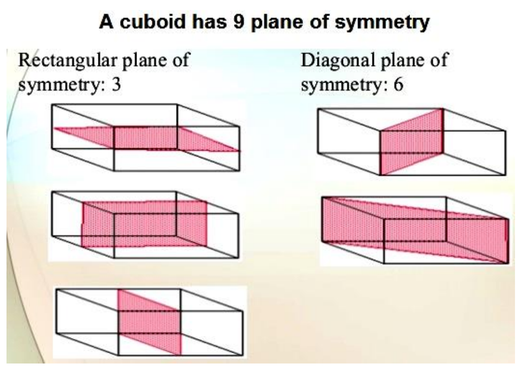
A plane through a solid that divides the solid into two parts that are mirror images of each other is called plane of symmetry.



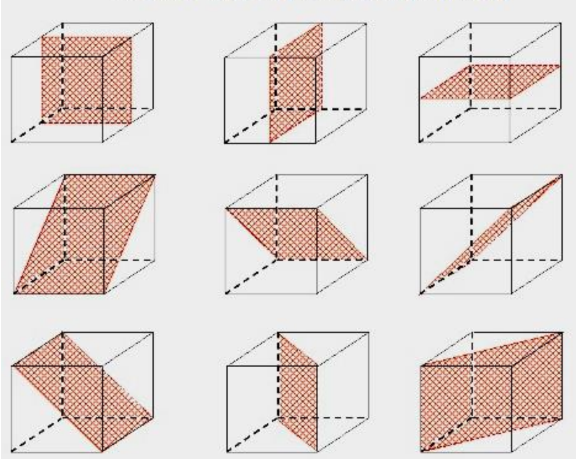




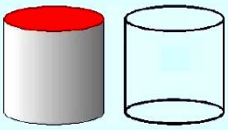




A cube has 9 plane of symmetry



Cylinder

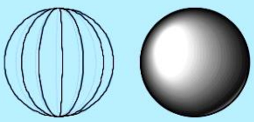


Key Feature
A prism with a circular cross-section.

Faces, Corners and Edges
The normal definitions of faces, corners and edges are not appropriate for a cylinder.

Planes of Symmetry?
Infinite

Sphere

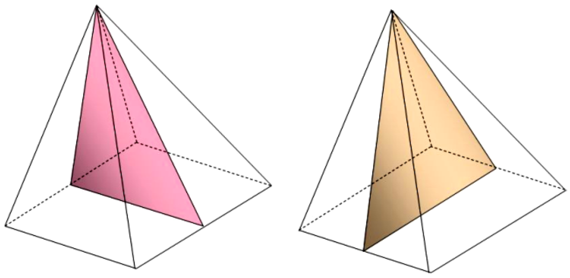


Key Feature
Every point on the surface of the sphere is the same distance from the centre.

Faces, Edges and Corners
The normal definitions of faces, corners and edges are not appropriate for a sphere

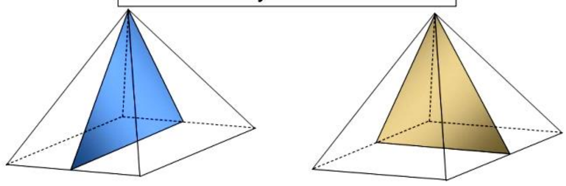
Planes of Symmetry?
Infinite

Pyramids

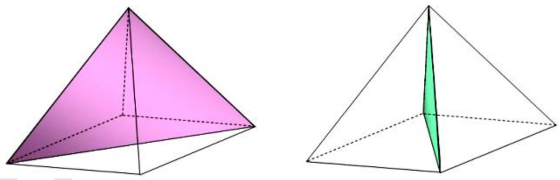


A rectangular based pyramid has 2 planes of symmetry.

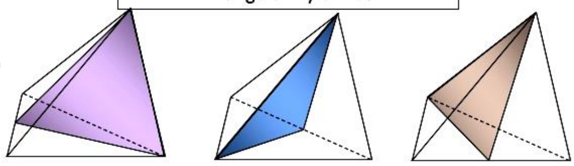
Pyramids



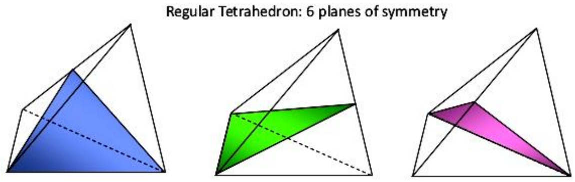
A square based pyramid has 4 planes of symmetry.



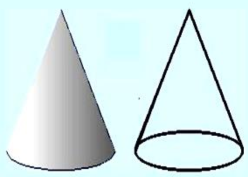
Triangular Pyramids



Regular Tetrahedron: 6 planes of symmetry



Cone

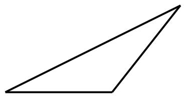
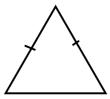
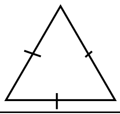
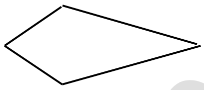
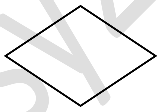
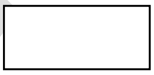
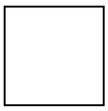
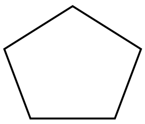
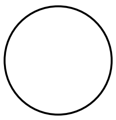
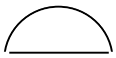


Key Feature
The point of the cone is directly above the centre of the circular base.

Faces, Corners and Edges
The normal definitions of faces, corners and edges are not appropriate for a cone.

Planes of Symmetry?
Infinite

Line of Symmetry and Order of Rotational Symmetry of Geometrical Figures

Name	Shape	No. of Line Of Symmetry	Order of Rotational symmetry
Scalene Triangle		0	1
Isosceles triangle		1	1
Equilateral Triangle		3	3
Isosceles Trapezium		1	1
Kite		1	1
Parallelogram		0	2
Rhombus		2	2
Rectangle		2	2
Square		4	4
Regular n-sided Polygon	 Pentagon	n=5	n=5
Circle		Infinite	Infinite
Semi Circle		1	1

Unit-17

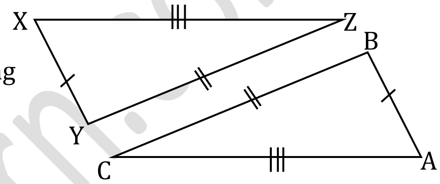
Similarity and Congruency

<u>Key Points</u>

- **Congruent Triangles:**
    - Triangles are congruent when all corresponding sides and interior angles are equal.
    - The shape and size of congruent figure is preserved under translation, rotation and reflection.

- **Test of Congruent Triangles**

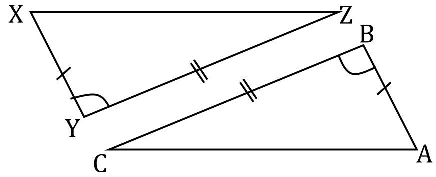
- **SSS Property**



The three sides of one triangle are equal to the corresponding three sides of the other triangle.

$In\ \Delta ABC\ and\ \Delta XYZ \quad AB = XY,\ BC = YZ\ and\ CA = YZ \quad Then\ \Delta ABC\ \equiv \Delta XYZ$

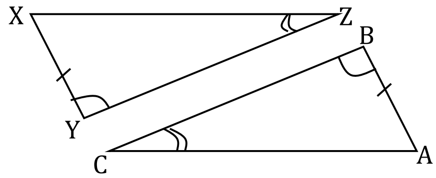
- **SAS Property**



Two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle

$In\ \Delta ABC\ and\ \Delta XYZ \quad AB = XY,\ BC = YZ\ and\ C\hat{B}A = Z\hat{Y}X\ \ Then\ \Delta ABC\ \equiv \Delta XYZ$

- **AAS Property**

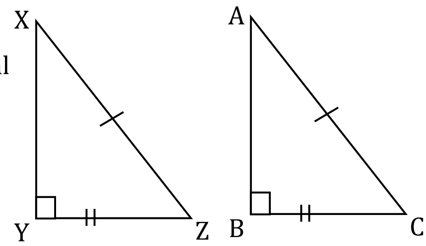


Two angles and a side of one triangle are equal to two angles and the corresponding side of the other triangle .

$In\ \Delta ABC\ and\ \Delta XYZ \quad AB = XY,\ A\hat{C}B = X\hat{Z}Y\ and\ C\hat{B}A = Z\hat{Y}X \quad Then\ \Delta ABC\ \equiv \Delta XYZ$

- **RHS Property**

The hypotenuse and one side of one right-angled triangle are equal to the hypotenuse and one side of the other right-angled triangle.

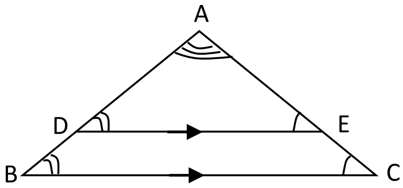


$In\ \Delta ABC\ and\ \Delta XYZ,\ AC = XZ\ (Hypotenouse),\ A\hat{B}C = XZY = 90^0\ and\ YZ = BC$

$Then\ \Delta ABC\ \equiv \Delta XYZ$

- **Similar Triangles:** Two triangles are similar if

(a) two angles of one triangle are equal to two corresponding angles of the other triangle;



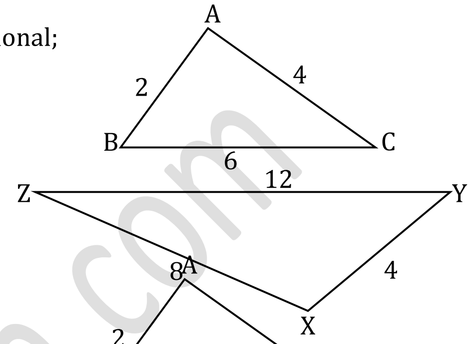
$In\ \Delta ABC\ and\ \Delta ADE$

$A\hat{B}C = A\hat{D}E$ (corrs. <s)

$A\hat{C}B = A\hat{E}D$ (corrs. <s)

$B\hat{A}C = B\hat{A}C$ $(common)$ $\Delta ABC\ is\ similar\ to\ \Delta ADE$

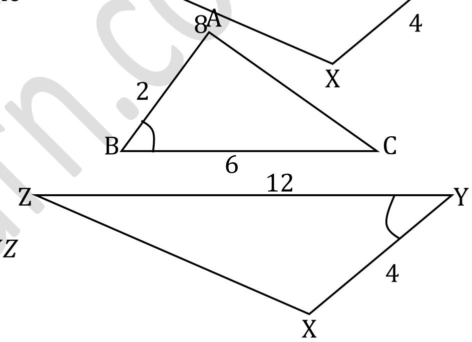
(b) all the corresponding sides of the two triangles are proportional;



In ΔABC and ΔXY $\frac{AB}{XY} = \frac{2}{4} = \frac{1}{2}$, $\frac{BC}{YZ} = \frac{6}{12} = \frac{1}{2}$ and $\frac{AC}{XZ} = \frac{4}{8} = \frac{1}{2}$

ΔABC is similar to ΔXYZ

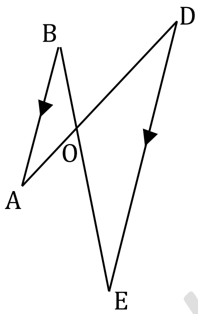
(c) An angle of one triangle is equal to the corresponding angle of the other triangle, and the sides which include the equal angle of both triangles are proportional.

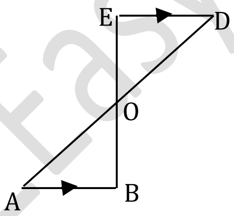


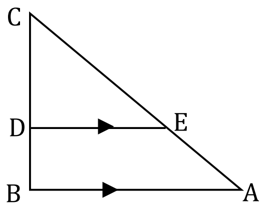
In ΔABC and ΔXYZ

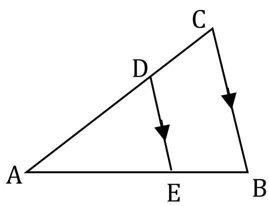
$\frac{AB}{XY} = \frac{2}{4} = \frac{1}{2}$, $\frac{BC}{YZ} = \frac{6}{12} = \frac{1}{2}$ and $A\hat{B}C = X\hat{Y}Z$, ΔABC is similar to ΔXYZ

Different Shapes of Similar Triangles:









$\Delta AOB\ is\ similar\ to\ \Delta DOE$ $\Delta ABO\ is\ similar\ to\ \Delta DEO$ $\Delta ABC\ is\ similar\ to\ \Delta EDC$ $\Delta ABC\ is\ similar\ to\ \Delta AED$

- **Similar Figures:**

- **Ratios of corresponding sides** $\frac{l_1}{l_2} = \frac{h_1}{h_2}$, $\frac{r_1}{r_2} = \frac{h_1}{h_2}$, $\frac{radius_A}{radius_B} = \frac{circumference_A}{circumference_B}$

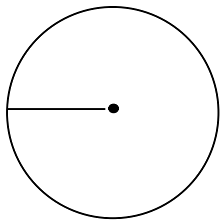
- **Ratio of areas** $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$, $\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$, $\frac{A_A}{A_B} = \left(\frac{circumference_A}{circumference_B}\right)^2$

- **Ratio of Volume or weight or capacity or mass** $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$, $\frac{W_1}{W_2} = \left(\frac{l_1}{l_2}\right)^3$, $\frac{m_1}{m_2} = \left(\frac{l_1}{l_2}\right)^3$

Unit-18

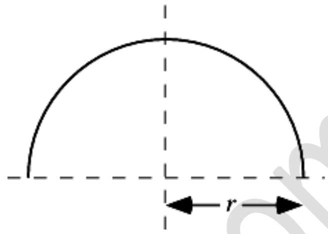
Mensuration

Key Points



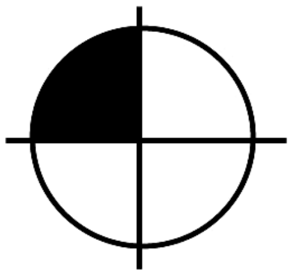
Area of circle $= \pi r^2$

Circumference of circle $= 2\pi r = 2d$



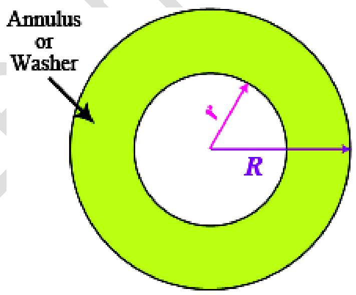
Area of Semi circle $= \frac{1}{2} \times \pi r^2$

Circumference of semi circle $= \frac{1}{2} \times 2\pi r$



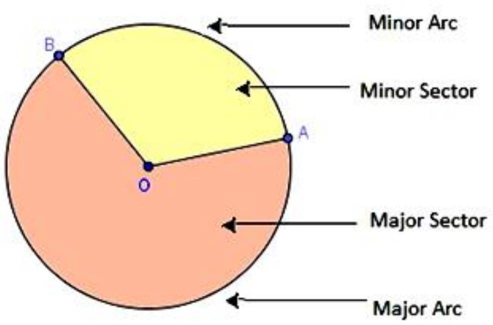
Area of Quad circle $= \frac{1}{4}\pi r^2$

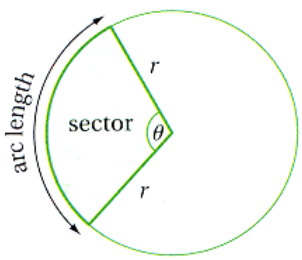
Circumference of Quad circle $= \frac{1}{4} \times 2\pi r$



Area of washer $= \pi R^2 - \pi r^2$

Perimetr of washer $= 2\pi R + 2\pi r$

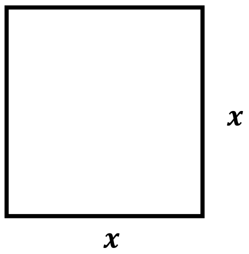




Area of sector $= \frac{\theta}{360} \times \pi r^2$

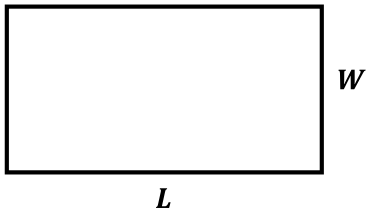
Length of arc $= \frac{\theta}{360} \times 2\pi r$

Perimetre of sector $= \frac{\theta}{360} \times 2\pi r + 2r$



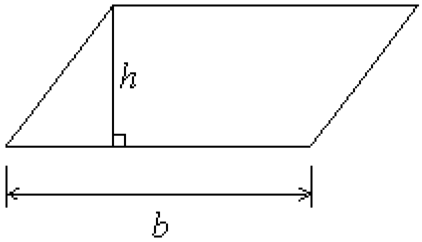
Area of Square $= x^2$

Perimeter of Square $= 4x$



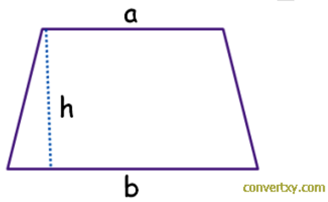
Area of Rectangle $= L \times W$

Perimeter of Rectangle $= 2(L + W)$



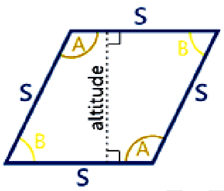
$$Area\ of\ Parallelogram = b \times h$$

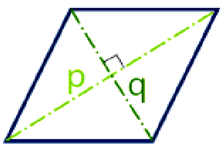
$$Perimetr\ of\ Parallelogram = sum\ of\ all\ sides$$



$$Area\ of\ Trapezium = \frac{1}{2}\ h\ (a + b)$$

$$Perimetr\ of\ Trapezium = sum\ of\ all\ sides$$



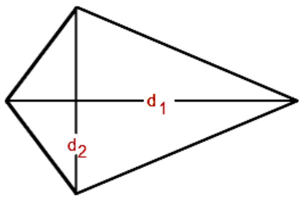


Area of rhombus $= h \times s$

Or

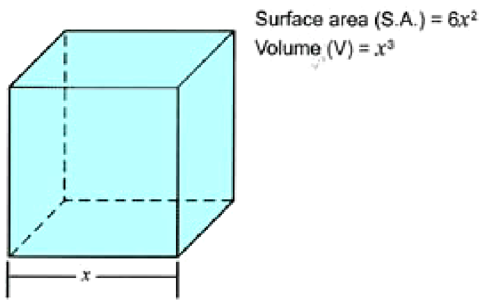
Area of rhombus $= \frac{p \times q}{2}$

Perimeter of rhombus $= 4\ s$



Area of kite $= \frac{d_1 \times d_2}{2}$

Perimeter of kite $=$ sum of sides

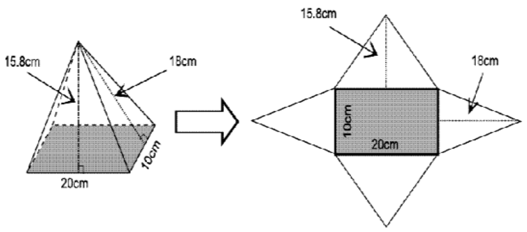


SUFACE AREA AND VOLUME OF CUBOID

cuboid

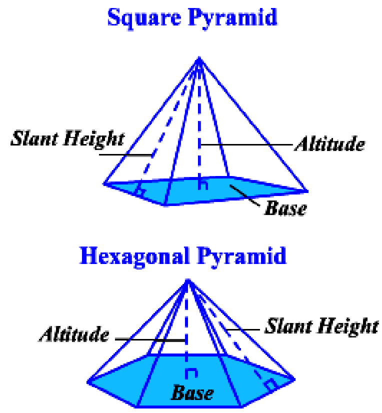
- Total Surface Area: 2(lb+ bh +hl)
- Lateral Surface Area:2h(l + b)
- Volume: l x b x h

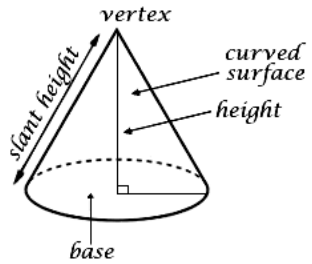
Pratima Nayak



Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height

Surface area of pyramid
$=$ Base Area
$+$ Area of all triangles

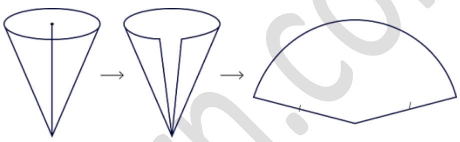




Volume of cone $= \frac{1}{3} \times \pi r^2 h$

Curved Surface of cone $= \pi r l$

Total Surface of cone $= \pi r l + \pi r^2$



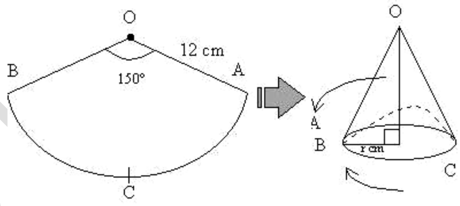
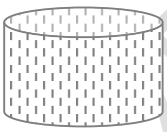


Figure 6a

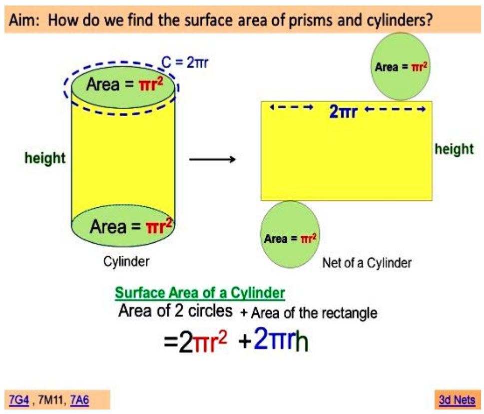
Figure 6b

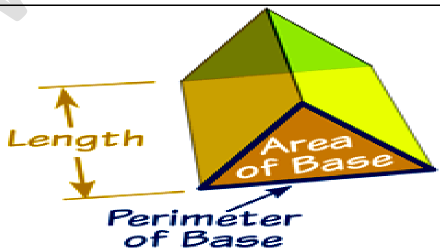


Volume of cylinder $= \pi r^2 h$

Curved Area of cylinder $= 2\pi r h$

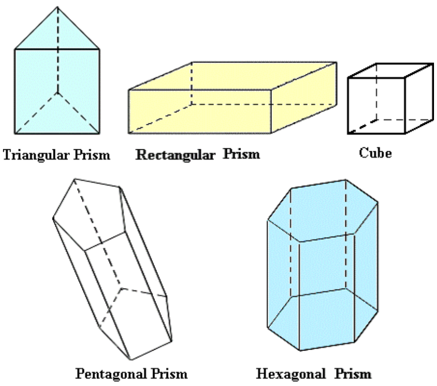
Total surface area of cylinder without top
$= 2\pi r h + \pi r^2$

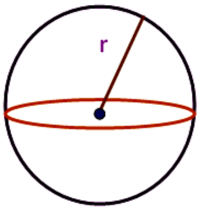




Volume of prism $=$ base area $\times$ height

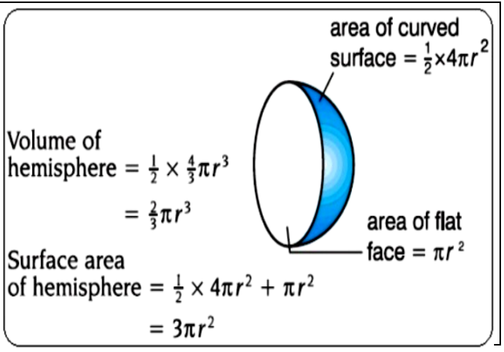
Surface Area of prism $=$
(perimeter of base $\times$ h) $+ 2 \times$ Base area





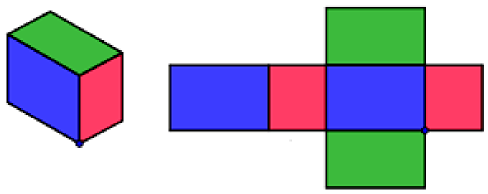
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

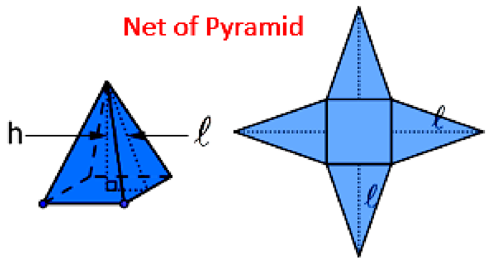


Nets of Solids

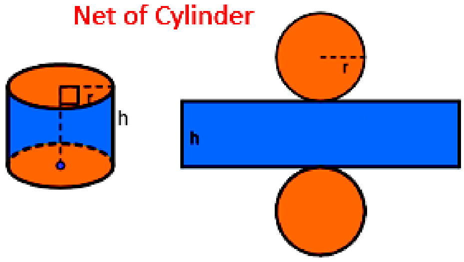
Net of Rectangular Prism



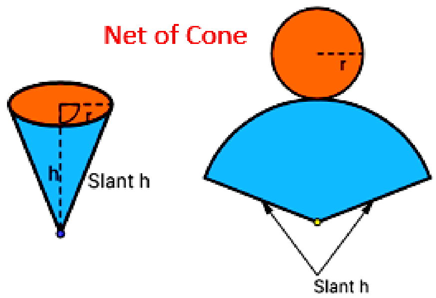
Net of Pyramid

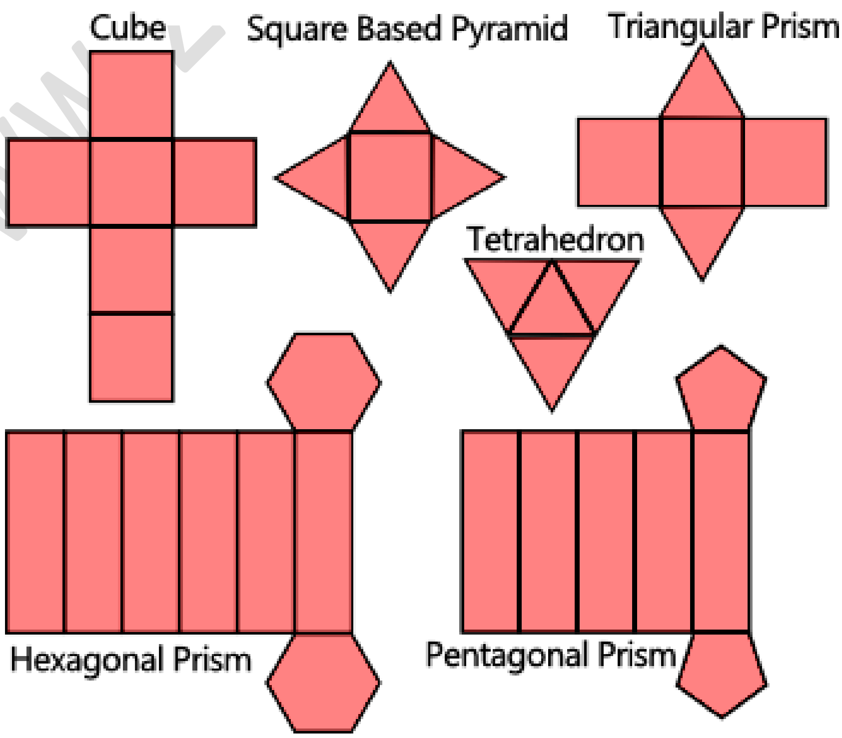


Net of Cylinder

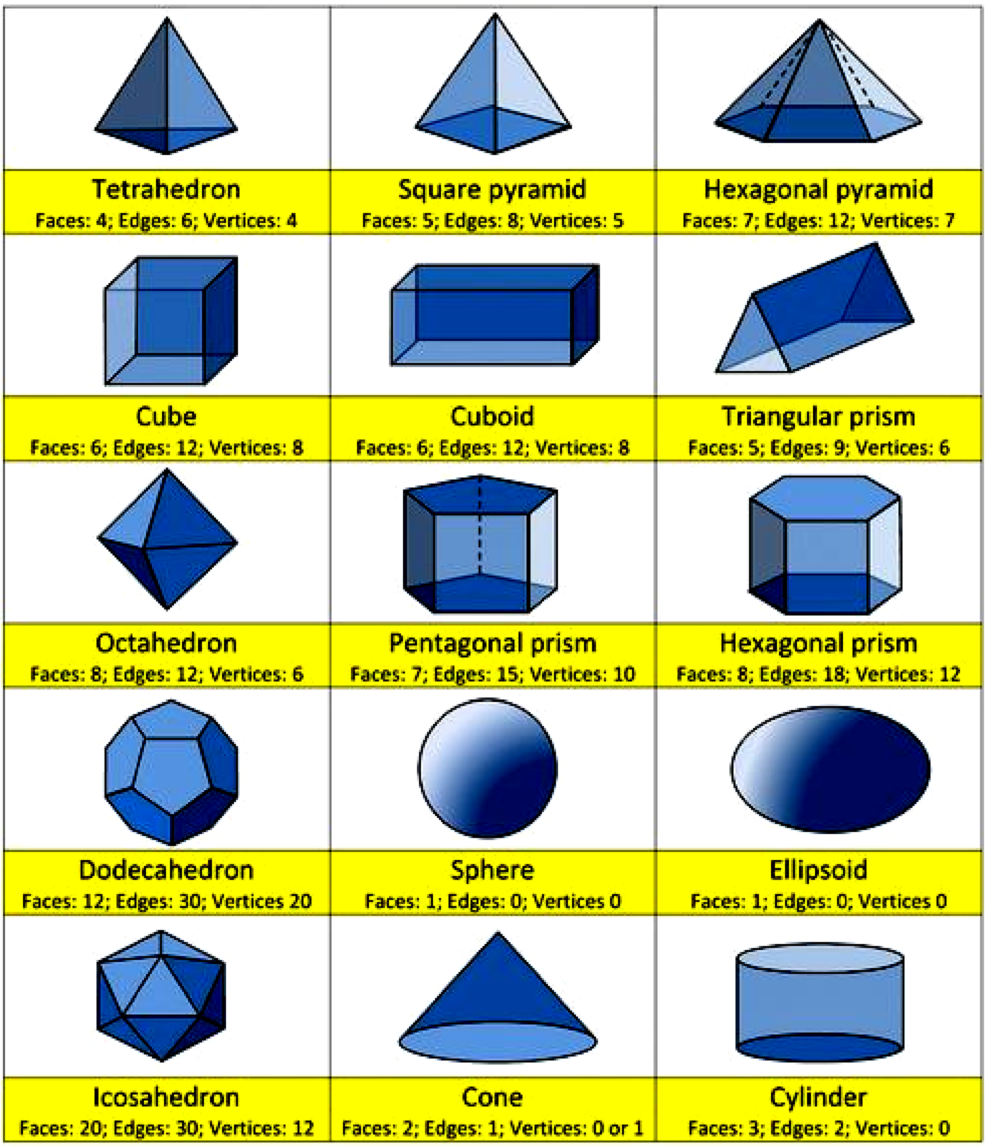


Net of Cone





Faces, Edges and Vertices of 3D Shapes



<u>Rate of Flow:</u>

Flow rate is the amount of fluid flowing in the given time. It is expressed in m^3/s or litre/second.

It is given by $\mathbf{Q = A\,V}$

where Q = rate of flow in m^3/s, A = cross-sectional are of channel, V = Speed of fluid.

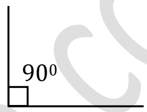
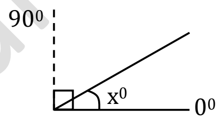
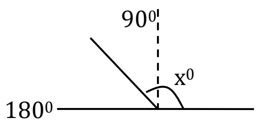
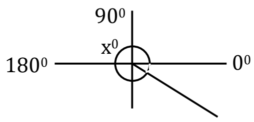
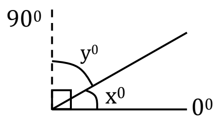
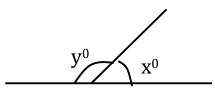
It is also given by $\mathbf{Q} = \frac{\textbf{Volume}}{\textbf{Time}}$

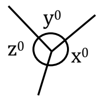
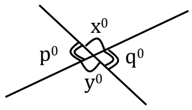
Unit- 19

Angle Properties of Parallel Lines and Polygons

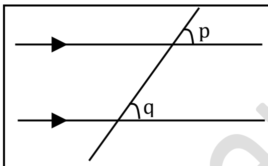
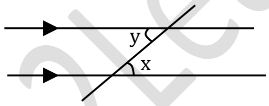
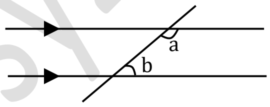
<u>Key points</u>

- **Geometrical Properties of Angles**

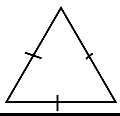
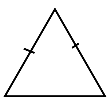
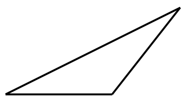
Name	Definition	Shape
Right Angle **Abbreviation: rt. <**	An angle that is exactly 90^0. $x = 90^0$	
Acute Angle **Abbreviation: act <**	Any angle that is less than 90^0. $0^0 < x < 90^0$	
Obtuse Angle **Abbreviation: obt. <**	Angle between 90^0 and 180^0. $90^0 < x\ 180^0$	
Reflex Angle **Abbreviation: ref. <**	Angle between 180^0 and 360^0. $180^0 < x < 360^0$	
Complementary Angles **Abbreviation: comp. <s**	Two or more angles who's sum is 90^0. $<x + <y = 90^0$	
Supplementary Angles **Abbreviation: <s on str. line**	Two or more adjacent angles on a straight line who's sum is 180^0. $<x + <y = 180^0$	

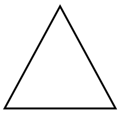
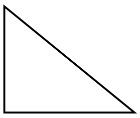
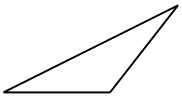
Angles at a point	The sum of angles at a point is 360^0. $<x + <y + <z = 360^0$	
Vertically Opposite Angles **Abbreviation: vert. opp. <s**	Vertically opposite angles are equal. $<x = <y$ and $<p = <q$	

- **Angle Properties of Parallel Lines**

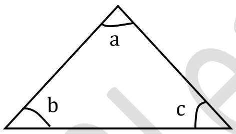
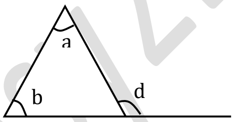
Name	Shape	Property
Corresponding Angles **Abbreviation: corres. <s**		$< p = <q$
Alternate Angles **Abbreviation: alt. <s**		$< x = <y$
Sum of Interior angles of Parallel lines **Abbreviation: Int. <s of // lines**		$< a + < b = 180^0$

- **Types of Triangles**

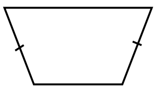
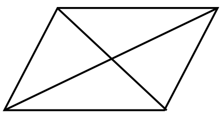
Name	Shape	Properties
Equilateral Triangle		➢ All sides are equal. ➢ All angles are of 60^0.
Isosceles triangle		➢ Two sides are equal. ➢ Two base angles are equal. ➢ $<x=<y$
Scalene Tiangle		➢ No equal side. ➢ All angles are different in size.

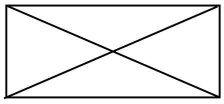
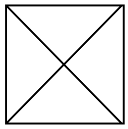
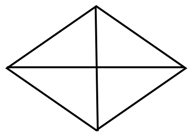
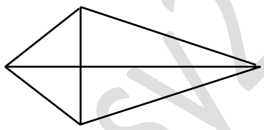
Acute Angled Triangle		➢ All three angles are acute.
Right angled Triangle		➢ One angle is of 90^0.
btuse Angled Triangle		➢ One angle is obtuse.

➢ **Angle Properties of Triangle**

Name	Shape	Properties
Interior Angles of a Triangle		➢ Sum of int. <s of triangle is 180^0. ➢ <a + < b + < c = 180^0
Exterior Angle of Triangle		➢ The ext. < of a triangle is equal to sum of two opposite int. <s ➢ <d = <a + <b
Sides and Opposite angles		➢ The longest side has opposite largest angle . ➢ The shortest side has opposite smalles angle.

➢ **Properties of Quadrilaterals**

Name	Shape	Properties
Trapezium		➢ Two opposite sides are parallel. ➢ Two opposite sides are non-parallel.
Isosceles Trapezium		➢ Two opposite sides are parallel. ➢ Two opposite sides are non-parallel but equal in size. ➢ Two pairs of equal angles.
Parallelogram		➢ Opposite sides are parallel and equal in size. ➢ Opposite angles are equal in size.

		➢ Diagonals are not equal in length. ➢ Diagonals bisect each other.
Rectangle		➢ Opposite sides parallel and equal in size. ➢ All corners angle are right angled. ➢ Diagonal are equal in size. ➢ Diagonal bisect each other.
Square		➢ Opposite sides are parallel. ➢ All sides are equal in size. ➢ All corners angle are right angled. ➢ Diagonal are equal in size. ➢ Diagonal bisect each other at 90^0.
Rhombus		➢ Opposite sides are parallel. ➢ All sides are equal in size. ➢ Opposite angles are equal in size. ➢ Corner angles are not right angled. ➢ Diagonal are not equal in size. ➢ Diagonal bisect each other at 90^0.
Kite		➢ No parallel side. ➢ Adjacent sides are equal in size. ➢ One pair of opposite angles are equal in size. ➢ The longer diagonal bisects the other diagonal at 90^0.

➢ **Angle Properties of n-sided Polygon**

- ❖ Sum of interior angles of a polygon = $(n - 2) \times 180^0$, where n is number of sides.
- ❖ Int. Angle of a regular Polygon $= \frac{(n-2)\times 180}{n}$
- ❖ Sum of exterior angles of a polygon $= 360^0$.
- ❖ Ext. Angle of a regular polygon $= \frac{360}{n}$
- ❖ Int. $<$ +Ext. $<$ = 180

➢ **Name of some common Polygons**

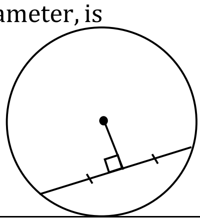
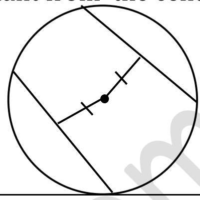
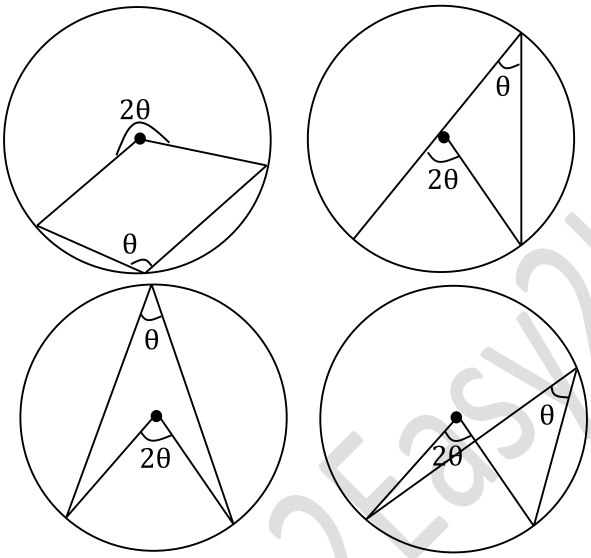
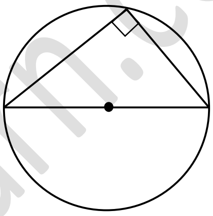
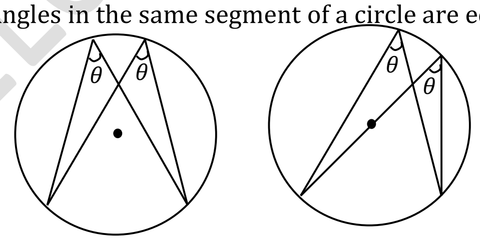
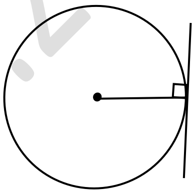
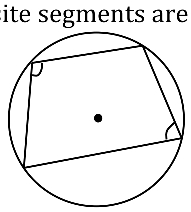
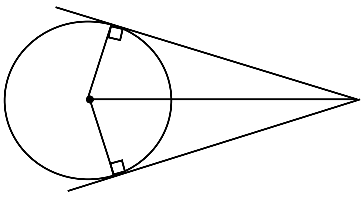
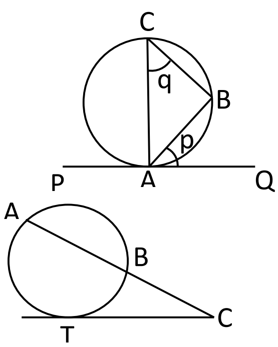
No. of sides	3	4	5	6	7	8	9	10
Name of polygon	triangle	quadrilateral	pentagon	hexagon	heptagon	octagon	nonagon	decagon

Unit- 20

Angles and Symmetry Properties of Circle

<u>Key Points</u>

- **Geometrical Properties of Circles**

A straight Line drawn from the centre of a circle to bisect a chord, which is not a diameter, is perpendicular to the chord. 	In equal circles (or in the same circle), equal chords are equidistant from the centres (or centre). 
An angle at the centre of a circle is twice that of any angle at the circumference subtended by the same arc.  	Every angle subtended by the diameter of a circle at the circumference is a right angle.  Angles in the same segment of a circle are equal. 
A tangent to a cirle is perpendicular to the radius drawn to the point of contact. 	Angles in opposite segments are supplementary 
Tangents drawn to a cirle from an external point are equal; they subtend equal angles at the centre, and the line joining the external point to the centre of the circle bisects the angle between the tangents. 	**Alternate Theorem:** The angle between the tangent and the chord through the point of contact is equal to the angle subtended by the chord in the alternate segment. $Q\hat{A}B = A\hat{C}B$ and $TC^2 = AC \times BC$ 

Unit-21

Locus and Constructions

Key Points

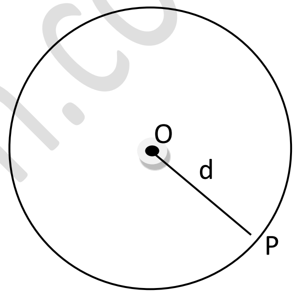
Locus:

In geometry, a **locus** (Latin for "place", plural *loci*) is a collection of points which share a property. For example, a circle may be defined as the locus of points in a plane at a fixed distance from a given point. A locus may alternatively be described as the path through which a point moves to fulfill a given condition or conditions. For example, a circle may also be defined as the locus of a point moving so as to remain at a given distance from a fixed point.

Loci In Two Dimensions

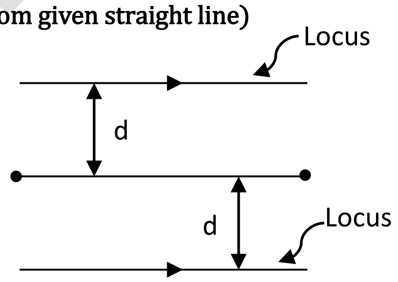
➢ **Circle: (Locus of a point at given distance from a fixed point)**

In general, the locus of a point P which is at a given distance d from a given point O is a circle with centre O and radius d.



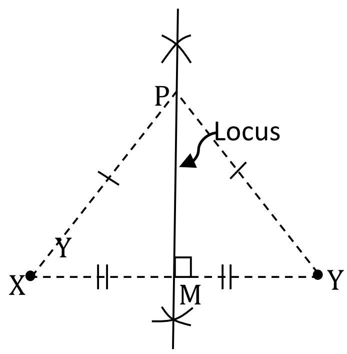
➢ **Parallel Lines: (Locus of a point at given distance from given straight line)**

In general, the loci of point which is at a given distance d from a given straight line XY are two straight lines parallel to XY and at a distance d from XY.



➢ **Perpendicular Bisector Of Line: (Locus of a point equidistant from two given points)**

In general, the locus of a point which is equidistant from two given points X and Y is the perpendicular bisector of the line XY.



How to draw Perpendicular Bisector

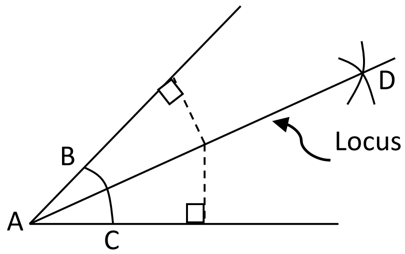
- Place the compass at one end of line.
- Adjust the compass to slightly longer than half the line length
- Draw arcs above and below the line.
- Keeping the same compass width, draw arcs from other end of line.
- Place ruler where the arcs cross, and draw the line.

- **Angle Bisector: (Locus of points equidistant from two given intersecting lines)**

  In general, the locus of a point which is equidistant

  From two given intersecting straight lines is a straight

  Line which bisects the angle between the two given lines.



  <u>**How to draw Angle Bisector:**</u>

  - Place the compass on the angle point A.
  - Draw an arc BC which intersects both arms of angle.
  - Place compass on point B and draw an arc.
  - Place compass (compass opening must not be changed) on point C and draw one more arc.
  - Both arcs will intersect at point D.
  - Draw a straight line by joining point A and point D. The line AD will be the angle bisector.

- **Intersections of Loci :**

  If two or more loci intersect at a point P, then P satisfies the conditions of the loci simultaneously.

- **Locus of a point for which area of given triangle remain constant.**

  The locus of a point P which moves such that the area of ΔABP remains constant is a set of points of two lines *l* and *m* parallel to and equidistant from *AB*.

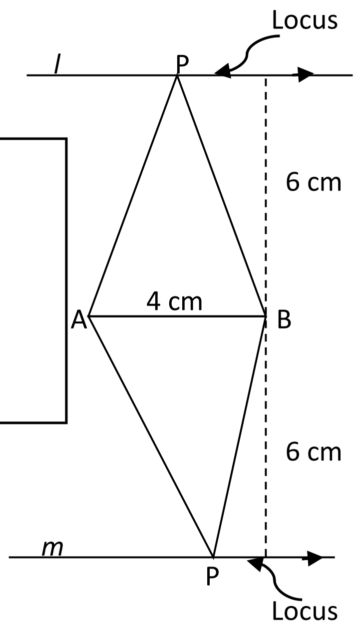
  If AB = 4 cm and area of triangle is 12 cm², then the locus of point P is shown in figure.

  Area of ΔABP = 12 cm²

  Area = ½ Bxh

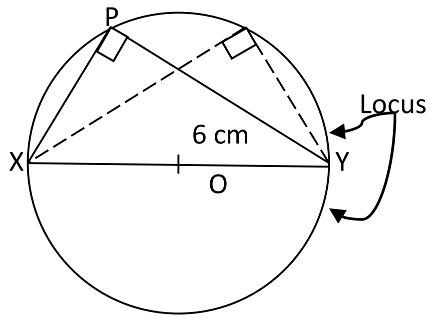
  12 = ½ x 4 x h

  h = 6 cm



- **Locus of points at which a line segment subtends an angle of 90⁰**

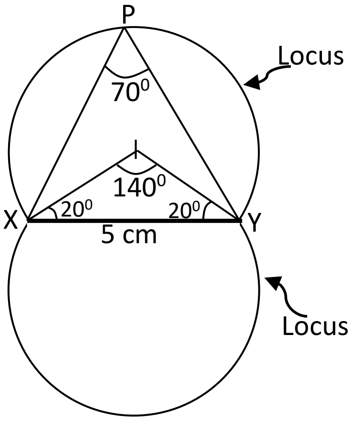
  Give that a line XY = 6 cm such that $X\hat{P}Y = 90^0$, then the locus of P is the set of points excluding X and Y, on the circumference of the circle with XY as diameter.



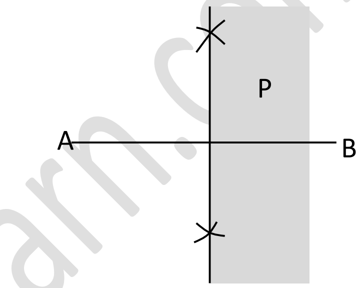
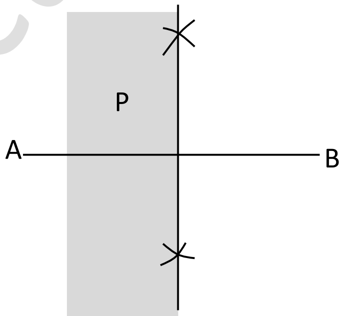
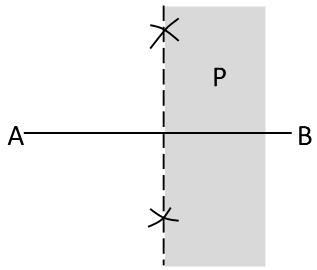
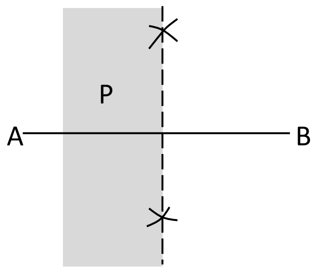
- **Locus of points at which a line segment subtends a given angle**

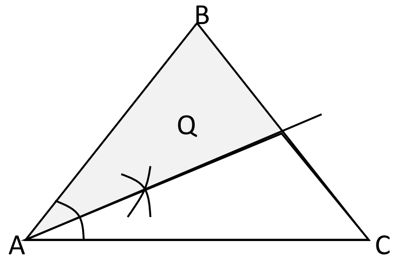
  Given that a line XY 5 cm, $X\hat{P}Y = 70^0$, the locus of the point P the set of the points on the arc XPY and its reflection in XY, excluding X and Y ,

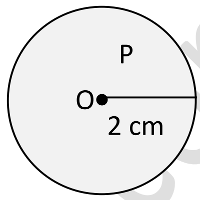
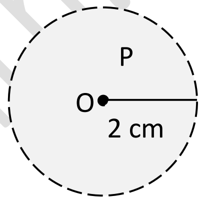
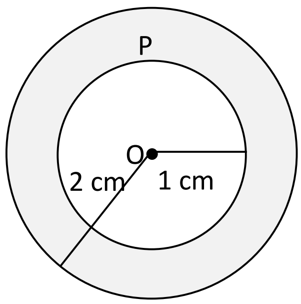
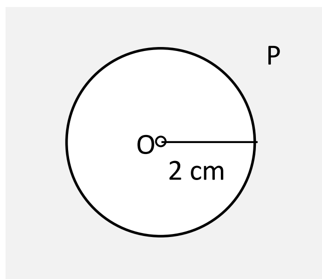
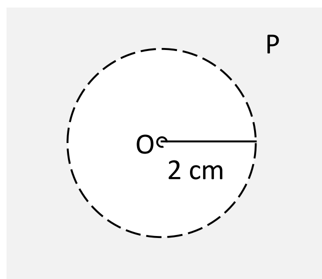
  $X\hat{O}Y = 2 \times 70^o = 140^0$ and $X\hat{Y}O = 20^0$



<u>Shading of Loci</u>

Sr.	Condition	Shading
1	AP $\geq$ PB	
2	AP $\leq$ PB	
3	AP $> PB$	
4	AP $< PB$	

5	$B\hat{A}Q \le C\hat{A}Q$	

Sr.	Condition	Shading
6	OP $\le$ 2 cm	
7	OP $< 2\ cm$	
8	1 cm $\le$ OP $\le$ 2 cm	
9	OP $\ge$ 2 cm	
10	OP $> 2\ cm$	

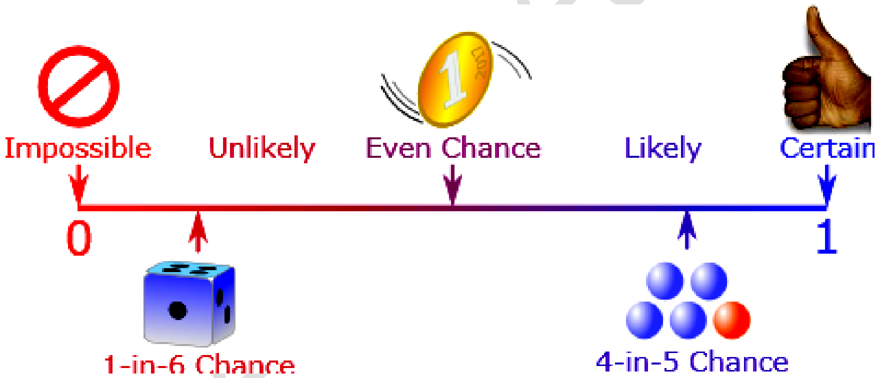
Unit-22

Probability

Key Points:

- Probability is the measure of occurrence of an event.
- If the event is impossible to occur then its probability is **0**.
- If the occurrence is certain then the probability is **1**.
- The closer to 1 the probability is, the more likely the event is.
- The probability of occurrence of an event E (called its success) will be
- denoted by P(E).
- $$\mathbf{P(E)} = \frac{\textbf{Number of favourable outcomes for Event E}}{\textbf{Numbers of Possible outcomes}} = \frac{\mathbf{n(E)}}{\mathbf{n(S)}} = \frac{\mathbf{K}}{\mathbf{m}}$$

- If an event has no outcomes, that is as a subset of S if E = ; then P(;) = 0.
- On the other hand, if E = S then P(S) = 1.
- When the outcome of an experiment is just as likely as another, as in the example of tossing a coin, the outcomes are said to be equally likely.
- Probability of event not happening = 1 – Probability of event happening



Experiment:
An **experiment** is any situation whose outcome cannot be predicted with certainty.
Examples of an experiment include rolling a die, flipping a coin, and choosing a card from a deck of playing cards.

Outcome Or Simple Event
By an outcome or simple event we mean any result of the experiment.
For **example**, the experiment of rolling a die yields six outcomes, namely, the outcomes 1,2,3,4,5, and 6.

Sample Space (S)
The sample space S of an experiment is the set of all possible outcomes for the experiment.
For **example**, if you roll a die one time then the experiment is the roll of the die. A sample space for this experiment could be S = {1, 2, 3, 4, 5, 6} where each digit represents a face of the die.

Event:
An event is a subset of the sample space.
For **example**, the event of rolling an odd number with a die consists of three simple events {1, 3, 5}.

Independent Events : Two events are independent if outcome of one event does not affect the probability of the outcome of the other event.

Examples: i) Rolling a die . ii) Tossing a coin.
iii) Drawing a card from a box and put it back before the second draw.

Dependent Events : Where an event **depends on** what happens in the previous event.

Example: Drawing a card from a box and does not put it back before the second draw. It is called without replacement. In this case the probability of second draw depends upon the first draw.

Addition of Probabilities and Mutually Exclusive Events ("OR" or Unions)

- **Mutually Exclusive Events**

Two events are mutually exclusive if they cannot occur at the same time. Another word that means mutually exclusive is disjoint.
If two events are disjoint, then the probability of them both occurring at the same time is 0.

Disjoint: P(A and B) = 0

If two events are mutually exclusive, then the probability of either occurring is the sum of the probabilities of each occurring.

- **Specific Addition Rule** *(Only valid when the events are mutually exclusive.)*

P(A or B) = P(A) + P(B)

Multiplication of Probabilities and Independent Events ("AND" or Intersections)

- **Independent Events**

Two events are independent if the occurrence of one does not change the probability of the other occurring.
An example would be rolling a 2 on a die and flipping a head on a coin. Rolling the 2 does not affect the probability of flipping the head.
If events are independent, then the probability of them both occurring is the product of the probabilities of each occurring.

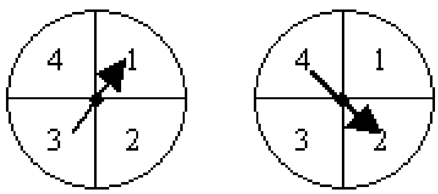
- **Specific Multiplication Rule** (*Only valid for independent events*)

P(A and B) = P(A) × P(B)

Probability Diagrams (or Possibility Diagrams)

- When an experiment is more complex constructing a probability diagram or possibility diagram may be helpful.
- In possibility diagram the outcomes from two events are displayed in a table.

Example:
The diagram shows two spinners, each of which is divided into 4 equal sectors. Each spinner has a pointer which, when spun, is equally likely to come to rest in any of the four equal sectors.



In a game, each pointer is spun once. Find the probability that
a) the pointers will stop at the same number b) the first spinner shows the larger number.

Solution: Construct the probability diagram. Each dot represents a possible outcome according to the coordinates.

a) Let A = event of getting the same number on the two spinners.

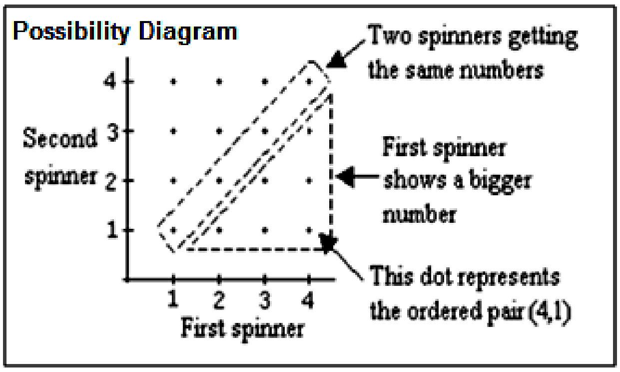
From the probability diagram, n(A) = 4, n(S) = 16

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{16} = \frac{1}{4}$$

b) Let B = event the first spinner shows the bigger number.

From the probability diagram, n(B) = 6

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{16} = \frac{3}{8}$$

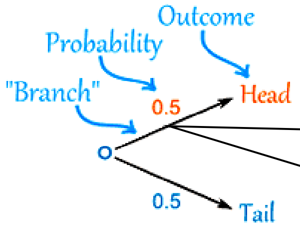


Probability Tree Diagrams

- Calculating probabilities can be hard, sometimes you add them, sometimes you multiply them, and often it is hard to figure out what to do ... **tree diagrams to the rescue!**

- Tree diagrams are really wonderful for figuring out dependent events (where an event **depends on** what happens in the previous event)

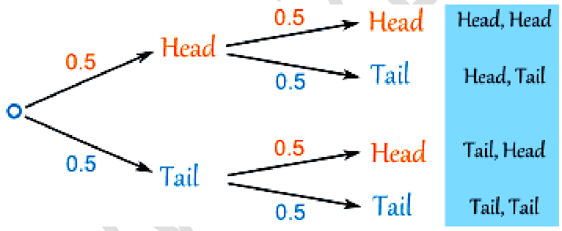
Here is a tree diagram for the toss of a coin:



There are two "branches" (Heads and Tails)

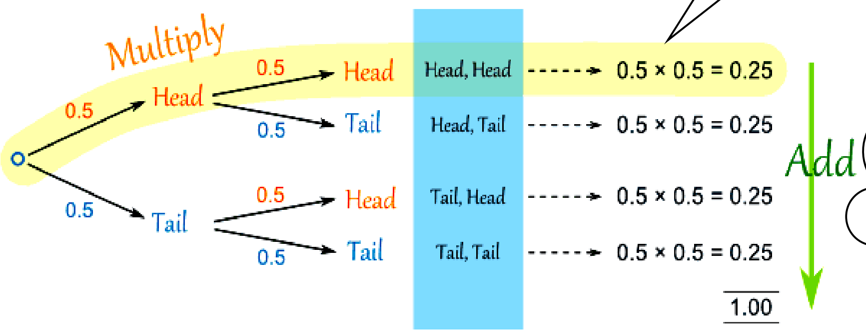
- **The probability of each branch is written on the branch.**
- **The outcome is written at the end of the branch.**

We can extend the tree diagram to two tosses of a coin:



- You **multiply** probabiliti es **along the branches.**
- You **add** probabilities down **columns.**

How do you calculate the overall probabilities?



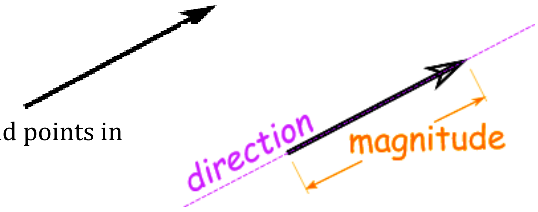
Conclusion: when in doubt draw a tree diagram, multiply along the branches and add the columns. Make sure all probabilities add to

Unit-23

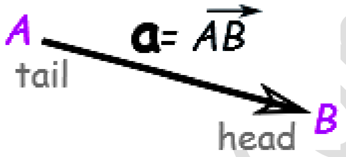
Vectors in Two Dimensions

Key Points

- **Vector**
  A vector has magnitude (size) and direction.
  The length of the line shows its magnitude and the arrow head points in the direction.



- **Vector Notation**
  A vector is often written in bold, like **a** or **b**.
  A vector can also be written as the letters of its head and tail with an arrow above it.

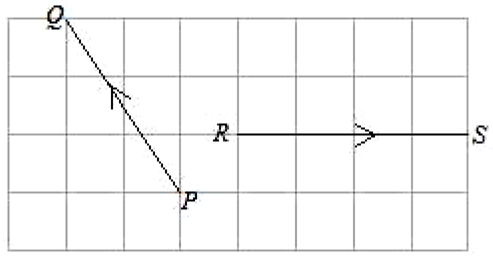


- **Column Vector**
  The vector can also be represented by the column vector $\begin{pmatrix} u \\ v \end{pmatrix}$. The top number is how many to move in the x-direction and the bottom number is how many to move in the y-direction.

- **Magnitude of a Vector**
  The length of a vector is called the magnitude.
  The magnitude of a vector is shown by two vertical bars on either side of the vector.

$$\text{If } \mathbf{a} = \begin{pmatrix} u \\ v \end{pmatrix}, \text{then } |\mathbf{a}| = \sqrt{u^2 + v^2}$$

Example



Solution:

$$\overrightarrow{PQ} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \text{ and } |PQ| = \sqrt{(-2)^2 + 3^2} = \sqrt{13} \text{ units}$$

$$\overrightarrow{RS} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \text{ and } |RS| = \sqrt{4^2 + 0^2} = 4 \text{ units}$$

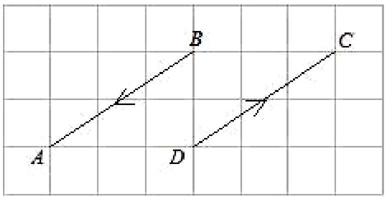
- **Zero vector:** A vector with magnitude 0 is called the zero vector.

- **Unit vector:** A vector with magnitude 1 is called a unit vector.

- **Negative vectors:** Vectors having the same magnitude but in the opposite direction.

  The negative vector of **a** is written as $-$ **a**.

Example A vector $\overrightarrow{AB}$ is a **negative vector** of $\overrightarrow{CD}$, if $\overrightarrow{AB}$ has the same magnitude as $\overrightarrow{CD}$ but are opposite in directions.



$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} -3 \\ -2 \end{pmatrix} = -\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

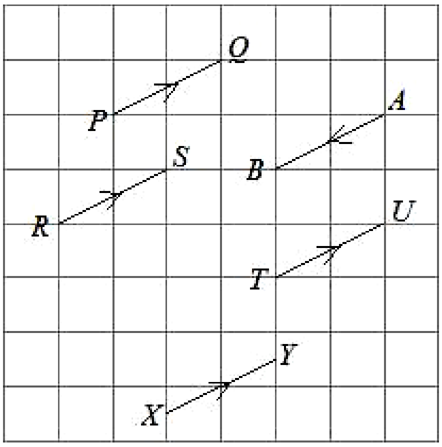
So, $\overrightarrow{AB} = -\overrightarrow{CD}$ and $-\overrightarrow{AB} = \overrightarrow{CD}$

$\overrightarrow{AB}$ and $\overrightarrow{CD}$ are negative vectors of each other.

- **Equal Vectors**

Equal vectors are vectors that have the same magnitude and the same direction.
Equal vectors may start at different positions. Note that when the vectors are equal, the directed line segments are parallel.

Example



All the vectors shown above are equal, except for $\overrightarrow{AB}$.

$$\overrightarrow{PQ} = \overrightarrow{RS} = \overrightarrow{TU} = \overrightarrow{XY} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \text{ but } \overrightarrow{AB} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

- **Equality Of Column Vectors**

If two vectors are equal then their vector columns are equal.

Example

The column vectors **p** and **q** are defined by $\mathbf{p} = \begin{pmatrix} 8-x \\ 6-y \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} x-4 \\ y+2 \end{pmatrix}$

Given that **p** = **q**, find the values of x and y

Solution $\mathbf{p} = \mathbf{q} \Rightarrow \begin{pmatrix} 8-x \\ 6-y \end{pmatrix} = \begin{pmatrix} x-4 \\ y+2 \end{pmatrix}$

$8 - x = x - 4$
$2x = 12$
$x = 6$

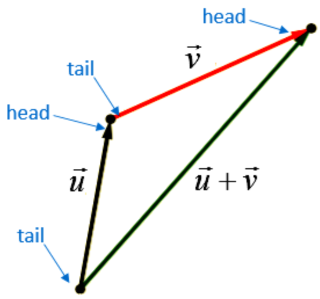
$6 - y = y + 2$
$2y = 4$
$y = 2$

- **Addition of Vectors**

The following diagrams show how to add vectors graphically using the **Triangle or Head-to-Tail** method and the **Parallelogram** method.

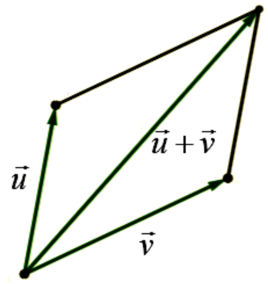
Graphical Methods for Vector Addition

Triangle Method or
Head-to-Tail Method



1. Place the vectors with the head of the previous vector $\vec{u}$ connected to the tail of the successive vector $\vec{v}$.
2. The resultant vector $\vec{u}+\vec{v}$ is formed by connecting the tail of the first vector to the head of the last vector.

Parallelogram Method

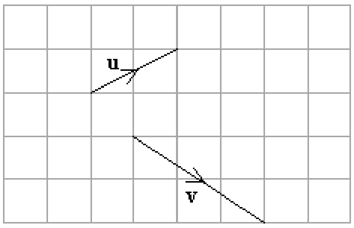


1. Place both vectors, $\vec{u}$ and $\vec{v}$ at the same initial point.
2. Complete the parallelogram.
3. The diagonal of the parallelogram is the resultant vector $\vec{u}+\vec{v}$.

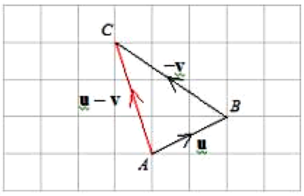
- **Subtraction of Vectors**

Subtracting a vector is the same as adding its negative.
The difference of the vectors **p** and **q** is the sum of **p** and **–q**.
p – q = p + (–q)

Example:
Subtract the vector **v** from the vector **u**.



Draw a vector $\overrightarrow{AB} = \mathbf{u}$. From the terminal point B, draw the vector $\overrightarrow{BC} = -\mathbf{v}$. Join A to C to complete the triangle.



Solution

u – v = u + (–v)
Change the direction of vector **v** to get the vector **–v.**

$\overrightarrow{AC} = \mathbf{u} - \mathbf{v}$

In terms of column vectors,

$\mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$$\begin{aligned} \mathbf{u} - \mathbf{v} &= \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 3 \end{pmatrix} \end{aligned}$$

- **Multiplying a Vector by a Scalar**

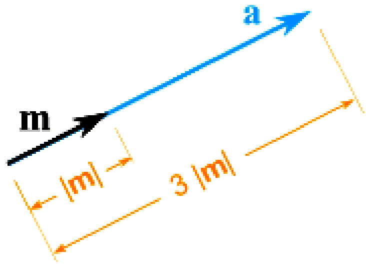
  When we multiply a vector by a scalar it is called "scaling" a vector, because we change how big or small the vector is.

  Example

  Multiply the vector $\boldsymbol{m} = \binom{7}{3}$ by the scalar 3.

  Solution

  $$\boldsymbol{a} = \boldsymbol{3m} = 3\binom{7}{3} = \binom{21}{9}$$



- **Parallel Vectors**

  Vectors are parallel if they have the same direction or in exactly opposite directions.

  Example: Given that $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ y \end{pmatrix}$ are parallel vectors, find the value of y.

  Solution:

  Since the given vectors are parallel, let

  $\begin{pmatrix} 8 \\ y \end{pmatrix} = k\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2k \\ -3k \end{pmatrix}$ where k is a scalar.

  $2k = 8 \Rightarrow k = 4$

  $y = -3k = -3(4) = -12$

- **How to define parallel vectors?**

  Two vectors are parallel if they are scalar multiples of one another.

  If **u** and **v** are two non-zero vectors and **u** = c**v**, then **u** and **v** are parallel.

- **Position Vector**

  A vector that starts from the origin (O) is called a position vector.

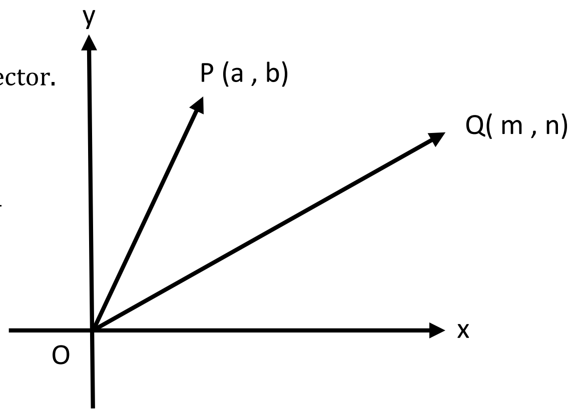
  P (a , b) is a point and its position vector is $\overline{OP} = \binom{a}{b}$

  Q (m , n) is a point and its position vector is $\overline{OQ} = \binom{m}{n}$

  The column vector $\overline{PQ}$ can be calculated by $\overline{PQ} = \overline{PO} + \overline{OQ}$

  Similarly we can find vectors if end points coordinates are given.

  Examples $\overline{XY} = \overline{XO} + \overline{OY}$, $\overline{AB} = \overline{AO} + \overline{\boldsymbol{O}B}$



- **Find the relation between vectors by using given ratio**

The line PQ is to be divided in the ratio 2 : 3 by the point R.

The relation between vectors can be written by using given ratio.

$\overrightarrow{PQ} = \frac{5}{2}\ \overrightarrow{PR}$, $\overrightarrow{PR} = \frac{2}{5}\ \overrightarrow{PQ}$, $\overrightarrow{RQ} = \frac{3}{2}\ \overrightarrow{PR}$,

Unit-24

Geometrical Transformation

Key Points

Transformations: Transformations are movements and changes of shape according to given rules. There are two types of transformations:

I. **Isometric Transformation**: shape and size remains the same but position/orientation changes. Reflection, Rotation and Translation are isometric transformations.
II. **Non-Isometric Transformation:** Shape/size and position changes. Enlargement, shear and stretch are non-isometric transformations.

Reflections (M)

A reflection, or flip, is a transformation that creates symmetry on the coordinate plane. You can use matrix multiplication to graph reflections in the coordinate plane.

i) Under reflection, an object and image are symmetrical about the mirror line. The object and image are exactly the same shape and size.

ii) Reflection does not preserve orientation. The object and image face in opposite direction.

iii) Points on mirror line are invariant. The image is as far behind the mirror line as the object is in front of it.

How to describe a reflection:

a) Write the name of transformation (Reflection).
b) Write the Equation of line of reflection.

How to draw a reflection:

a) Select a point on object and measure its distance (or count squares) from the line of reflection. The distance must be measure in the perpendicular direction to the mirror line.
b) Measure the same distance (or count squares) beyond the mirror line and mark the point.
c) Repeat the same procedure with all other points, and join all new points to get the image.

How to find the line of reflection: draw the line bisectors of two corresponding points of object and image. Both line bisectors overlap which shows that it is reflection otherwise it is rotation.

How to find equation of line of reflection, find midpoints of two corresponding points, let $A\acute{A}$ and $B\acute{B}$. Use these coordinates to find gradient and y-intercept. ($y = mx + c$)

Transformation	Line of reflection	Object	Image	Matrix
Reflection	x – axis	(a , b)	(a , – b)	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Reflection	y– axis	(a , b)	(– a , b)	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
Reflection	y = x	(a , b)	(b , a)	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Reflection	y = – x	(a , b)	(– b , – a)	$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Rotation (R):

A rotation is a transformation that turns a figure about a fixed point called a center of rotation. Rotation is defined by its center, angle and direction. The center of rotation is only invariant point.

How to identify rotation:

a) The object and image have same shape and size.
b) The object and image are facing different ways (but not flipped as in reflection)
c) All points on the object and image are at the same distance from the center of rotation.

How to describe the rotation

a) Write down the name of transformation (Rotation)
b) Write the coordinates of center of rotation, angle and direction of rotation.

How to rotate an object by using tracing paper

a) Place tracing paper on the object.
b) Trace one the axes and object on tracing paper.
c) Place the point of pencil on center of rotation.
d) Turn the tracing paper through the given angle and direction. The traced axes will guide here.
e) Mark the image from tracing paper on original paper.

How to rotate an object geometrically

a) Join the center of rotation with one of the point of object by dotted straight line.
b) Place the center of protractor on the center of rotation and zero towards the point.
c) Rotate the dotted line by marking the given angle in given direction (Clockwise or Anti clockwise).
d) Measure the distance between center of rotation and selected point on object.
e) Mark the point at the equal distance from the center of rotation on the rotated dotted line.
f) Repeat the same the same procedure for rest of the points.
g) Join the all new marked points to get rotated image.

How to find the center of rotation Draw line bisectors of two pairs of corresponding points of

Object and image. The point of intersection of two line bisectors is the center of rotation.

How to find angle and direction of rotation Join corresponding points of object and image with center of rotation and observe angle and direction.

Rotation by Matrices:

Transformation	Angle, direction , center	Object	Image	Matrix
Rotation	Rotation about origin, 90^0 Clockwise/ 270^0 Anticlockwise	(a , b)	(b , $-a$)	$R_{90C} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
Rotation	Rotation about origin, 90^0 Anti-Clockwise/ 270^0 Clockwise	(a , b)	($-$b , a)	$R_{90AC} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
Rotation	Rotation about origin, 180^0	(a , b)	($-$a , $-$b)	$R_{180} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Translation (T)

A translation moves all points of an object on a plane the same distance and in the same direction. Translation has no invariant point.

How draw a translation:

a) Select one point on object.
b) Count along horizontally the number of units shown on the top of the column vector.
c) Then count along vertically the number of units shown at the bottom of the column vector.
d) Repeat the same procedure for all point.
e) Join new marked points to get image.

How to identify a translation:

a) The object and image have same shape and size.
b) The direction (orientation) of object and image remains the same.

How to describe a translation:

a) Write the name of transformation (Translation).
b) Write down column vector$\binom{a}{b}$. The top number (a) shows horizontal displacement and bottom number (b) shows vertical displacement.

Translation by adding column vector

Translation	Translation under translation vector T=$\begin{bmatrix} a \\ b \end{bmatrix}$	Image = Object + T $\begin{bmatrix} \acute{x} \\ \acute{y} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}$	$\begin{bmatrix} \acute{x} \\ \acute{y} \end{bmatrix}$ *is image of* $\begin{bmatrix} x \\ y \end{bmatrix}$ *and* $\begin{bmatrix} a \\ b \end{bmatrix}$ *is translation vector*

Enlargement (E)

An enlargement makes an object larger or smaller according to a given scale factor.

i) An enlargement is defined by its center and scale factor. A scale factor is the ratio of length of the image to the corresponding length of the object.

ii) If scale factor is positive then object and image will be on one side of center of enlargement.

iii) If scale factor is negative, then center of enlargement will be in between object and image.

iv) Centre of enlargement is only invariant point.

v) Scale Factor (k) : $k = \frac{Length\ of\ image}{length\ of\ object}$

vi) $Area\ of\ Image = k^2 \times (area\ of\ object)$ or $\mathbf{A_I : A_O = k^2}$ or $\mathbf{\frac{A_I}{A_O} = k^2}$

How to identify an enlargement:

a) The image is smaller or larger than the object.
b) The shape and angles of image and objects remains the same but sides are proportional.
c) Position of image depends upon the scale factor.
d) If scale factor is positive then object and image is positioned on one side of centre of enlargement and image orientation remains the same.
e) If scale factor is negative then center of enlargement is positioned in between object and image. Image is also flipped.

How to find center of enlargement:

Join the corresponding points on object and image by straight dotted lines. These lines intersect at common point which will be the center of enlargement.

How to find scale factor:

Scale factor is the ratio of length of image to the corresponding length of the object.

$S.F = \frac{length\ of\ image}{length\ of\ object}$. Mention the sign($+\ or\ -$) of scale factor carefully.

How to describe an enlargement:

a) Write the name of transformation (Enlargement)
b) Write coordinates of center of enlargement.
c) Write scale factor

How to draw an enlargement geometrically:

a) Join the center of enlargement to each of the corners of the object by dotted line.
b) If S.F is positive, extend the lines in the direction of object.
c) If S.F is negative, extend the lines in the opposite direction of object.
d) Measure the distance between center and corner of the object.
e) Multiply this distance by scale factor and mark new distance from center on the dotted line.
f) Repeat the same procedure for rest of the corners and mark new points.
g) Join all new point to get image.

How to draw an enlargement by counting the blocks:

a) Count the numbers of blocks of a corner on object along (or back) and up (or down) with reference to the center of enlargement.
b) Multiply these distances by scale factor.
c) Mark these new distances by counting block with reference to the center.
d) Repeat the same procedure for the rest of the corners.
e) Join all new point to get image.

Matrix for Enlargement in the Coordinate Plane

$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ represents an enlargement matrix of scale factor k, with the origin as centre of enlargement

Combine Transformations

If M represents a reflection in the y-axis and R represents a 90^0 anticlockwise rotation about origin, then MR represents a 90^0 anticlockwise rotation about origin followed by a reflection in the y-axis RM represents a reflection in y-axis followed by a 90^0 anticlockwise rotation about origin

How to find the Matrix of given Transformation:

$\mathbf{M_I = M \times M_O}$

$\mathbf{M = M_I \times [M_O]^{-1}}$

Example: If $\begin{bmatrix} a & b \\ c & c \end{bmatrix}$ is a transformation matrix which maps $\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$ *onto* $\begin{bmatrix} 4 & 6 \\ 2 & 2 \end{bmatrix}$, find $\begin{bmatrix} a & b \\ c & c \end{bmatrix}$

Solution:

$\begin{bmatrix} a & b \\ c & c \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 2 & 2 \end{bmatrix}$, $\begin{bmatrix} a & b \\ c & c \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}^{-1}$

$$\begin{bmatrix} a & b \\ c & c \end{bmatrix} = \frac{\begin{bmatrix} 4 & 6 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -2 & 1 \end{bmatrix}}{-4} = \begin{bmatrix} 1 & 1.5 \\ 0 & 1 \end{bmatrix}$$

How to find Inverse Transformation Matrix: If P represent the transformation that maps the figure A onto the figure B, Then the transformation Q that will map figure B onto A is called inverse transformation of P, written as P^{-1}, ($i.e.\, \boldsymbol{Q = P^{-1}}$) .

Example:

If $\begin{bmatrix} 1 & 1.5 \\ 0 & 1 \end{bmatrix}$ is a matrix which maps object A onto object B then $\begin{bmatrix} 1 & 1.5 \\ 0 & 1 \end{bmatrix}^{-1}$ will be the matrix which maps object B onto object A.

Unit-25

Problem Solving and Pattern

Sequence: A sequence is simply a list of numbers which may be connected by a rule or pattern.

nth Term: The nth term is a formula with 'n' in it which enable us to find any term of a sequence without having to go up from one term to the next.

Even Numbers Sequence : 2, 4, 6, 8, ……..nth term = 2n

Odd Numbers Sequence: 1, 3, 5, 7, ………., nth term = 2n – 1

Square Numbers Sequence: 1, 4, 9, 16, ……, nth term = n^2

Cube Numbers Sequence: 1 , 8, 27, 64, …….., nth term = n^3

Triangle Numbers Sequence: 1, 3, 6, 10, 15, …….., nth term = $\frac{1}{2}$n (n + 1)

Constant Difference Sequence: Constant Difference Sequence is a sequence of numbers such that the difference between the consecutive terms is constant.

The nth term of the progression is given by $a_n = a + d\,(n - 1)$

Where a_n = nth term, a = first term, n= number of term d= constant difference

Changing Difference Sequence: Changing Difference Sequence is sequence of numbers such that the difference between the consecutive term is not constant but difference of difference is constant.

Example:

Term No.	1	2	3	4	5 ………………	n
Sequence	2	3	6	11	18 ……………	n^{th} term
Difference	*1*	*3*	*5*	*7*		
Difference of difference		*2*	*2*	*2*		

$a_n = a + d\,(n - 1) + \frac{c}{2}(n - 1)(n - 2)$

where a = first term, n = number of term, d = difference between first two numbers

c = difference of the difference

$a = 2 \quad d = 1 \quad c = 2$

Geometric Progression: A geometric progression is sequence of number in which each successive number is obtained by multiplying a fixed quantity with the previous number.

The nth term of geometric progression is given by $a_n = a\,r^{(n-1)}$

Where a_n = nth term , a = first term, n = number of term, r = common ratio